Material Selection for Spur Gear Design Using Ashby Chart

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Abstract - *This Study gives you a detailed and a comprehensive procedure of material to be selected for a Spur Gear Design. A highly efficient and advanced technique of material selection is used in this study, popularly called Ashby's Chart. Michael F. Ashby, after careful investigation of the material properties such as Strength, Density, Young's Modulus, etc. have formulated Ashby Charts. In these charts Mr. Ashby have plotted the material according to their properties. The basic Objective was set to Minimize the mass and Maximize the Strength. The Lewis Equation was used to determine the Bending Strength and to calculate the Material Index. Free variables and constraints are decided and thus the material is selected using Screening, Ranking and Documentation. A significant accuracy was found after implementing this technique as compared to the conventional methods.*

***Key Words*: Ashby Chart, Ashby Method, Optimization, Spur Gear Design, Screening, Ranking, Material Index, Lightweight.**

1. INTRODUCTION

Gears are usually defined as Toothed wheels or Multilobed cams, which transmit power and motion from one shaft to another by means of successive engagement and disengagement of teeth. Gears are commonly used in various industries in numerous machines, for instance in Factory automation, industrial robots, construction machines, automobiles, etc. Spur Gear have teeth parallel to the axis of rotation and are used to transmit power and motion from one shaft to another (parallel shafts). Among all of the types of gears, Spur Gear is considered to be the simplest one [2]. The design of spur gears depends on input parameters such as power, speed, operating conditions, fatigue life, and require iterative process. Many Researchers have analyzed and check with the help of computer aided engineering tools and therefore have estimated effective circumferential force on the tooth at the pitch circle of the gear while in meshing, there are effectively two kinds of stresses induced in the gear pair during the power and motion transmission from one shaft to another. They are (a) Bending stresses, induced on the gear teeth due to the Tangential force and (b) Surface contact stresses induced due to the radial component of the power being transmitted [4],[5].

A wide variety of steels, cast irons, bronzes, and phenolic resins have been used for gears. New materials such as nylon, titanium, and sintered iron have also become important in gear work [1]. Materials and the manufacturing processes that convert them into useful parts underlie all of engineering design. There are over 100,000 engineering materials to choose from. The typical design engineer should have ready access to information on 30 to 60 materials, depending on the range of applications he or she deals with [11]. Due to the rapid development in the material science field, more and more materials are being put forward by the researchers. This has caused a tremendous increase in the Material Universe and focused our attention on the competition between 6 broad classes: Metals, Polymers, Elastomers, Ceramics, Glasses, Composites and thus, has led to the confusion in the process of Material selection.

A technique suggested by Michael Ashby is an advanced material Selection process which provides Material Chart to obtain the optimum material for the desired Objective such as Maximizing Mass or Stiffness. Material limit performance and thus this technique display the idea of plotting one property against another. If the technique is meticulously implemented it gives us the potential candidate materials for the selection process [6]. The proposed idea could be readily implemented as computer aided tool, on the CES EDUPACK software. The properties like Mechanical, Optical, Thermal, Physical are all given emphasis on in the Ashby Charts [7].

Nowadays the Lightweight and High Strength Designs are required for nearly every application, for instance Automobiles, Robotic application, Aerospace Industry and Machinery. In this study we would investigate the materials required for designing of Lightweight and High Strength Spur Gear. The main Objectives, Design requirements, Constraints and free variables were found out. Ashby Charts were used to find out the potential candidate materials with the help of the Material Index (Screening). Later the materials were ranked according to the design requirement and thus the desired material could be found out.

2. Material Selection Strategy

Selection of a material is a very vital and significant step in the process of Designing (Spur Gear in this case). If a material is incorrectly chosen it can lead to not only failure but also will add up to the manufacturing cost. Selecting the best material for the part involves choosing a material which would fulfil the desired requirements and retaining all of its properties in the manufactured part. And thus, a prominent Selection strategy is required. The following Figure displays the Strategy implemented for this study.

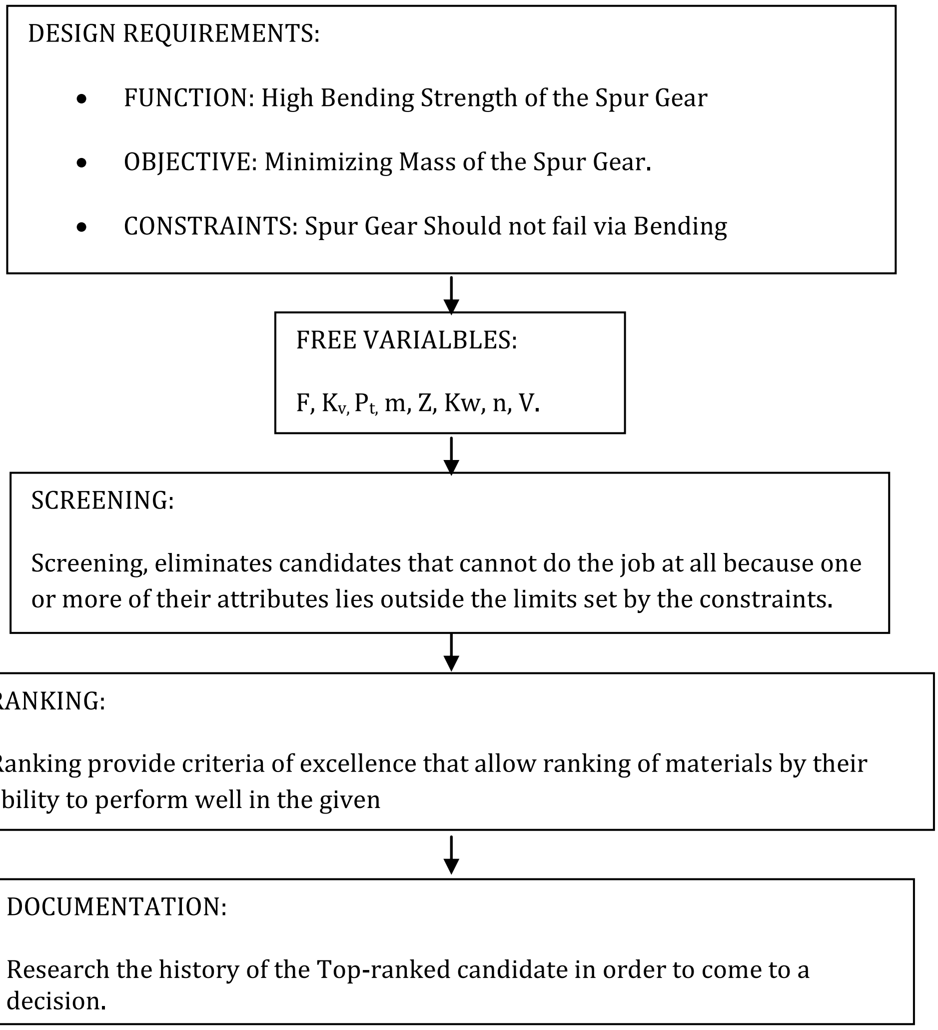

Figure 1.0: Material Selection Strategy.

The Material Index is found out after a careful evaluation of the constraints, objectives, function and free variables (the material index is evaluated in the later part of this study). The Performance of the structural element is determined by three things as per Ashby Method: The function Requirement, The Geometry and the properties of the material of which it is made [6]. It is best Described by the following equation:

$$P = \left[\begin{pmatrix} \text{Functional} \\ \text{requirements}, F \end{pmatrix}, \begin{pmatrix} \text{Geometric} \\ \text{parameters}, G \end{pmatrix}, \begin{pmatrix} \text{Material} \\ \text{properties}, M \end{pmatrix} \right] \quad \ldots\ldots (1)$$

Or $P = f(F, G, M)$

2.1 OBJECTIVE OF THE SPUR GEAR

The Main Objective of the Design is to Minimize Mass of the Spur Gear. In order to do that we have to primarily seek the equation describing the Mass of the Spur Gear. The Mass of the Spur Gear can be calculated by considering the Spur Gear as a Cylinder with height equal to Face width of the Spur Gear and radius equal to the half of the Pitch Circle Diameter of The Spur Gear [8].

Therefore, the Mass of the Spur Gear is given by: -

$$M = \pi \left(\frac{PCD}{2}\right)^2 F \rho \quad \ldots\ldots (2)$$

And,

$8m \le F \le 12m \rightarrow F = C_1 m$, (where: $8 \le C_1 \le 12$;) ……. (3)

$PCD = m Z$ ……. (4)

The Mass equation (2) becomes:

$$M = \left(\frac{\pi Z^2}{4}\right) C_1 m^3 \rho \quad \ldots\ldots (5)$$

2.2 CONSTRAINTS OF THE SPUR GEAR

There are two ways of designing the Spur Gear: (a) Bending Strength, (b) Wear Strength. If the wearing constraint of the gear is of supreme importance then the Spur Gear should be designed using the pitting resistance, but in this study the Spur Gear is Designed using the Bending Stress Fatigue criteria because it is the primary requirement in the industry. According to the bending Strength theory, the fatigue Strength (S) of the Spur Gear should be greater than or equal to the Bending Stress on the Spur Gear teeth.

$$\rightarrow \sigma \le S \quad \ldots\ldots (6)$$

According to the Lewis Theory, Spur Gear must be able to withstand the Tangential Load (P_t). The Lewis Bending stress equation is used as it is fairly simple to calculate the Material Index of the Spur Gear using it [9]. The Tangential force according to the Lewis theory is shown below:

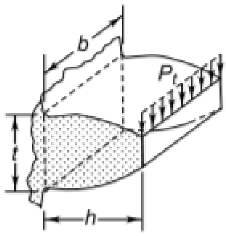

Figure 2.0: Gear tooth as a Cantilever Beam

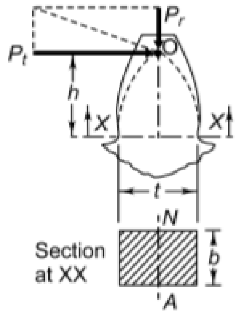

Figure 3.0: Gear tooth as parabolic beam

(NOTE: In this figure Face width is designated as b, but we will continue to use F as Face width)

From the Lewis equation stress can be deduced as:

➔ $$\sigma = \frac{Kv\ Pt}{F\ m\ Y} \qquad \text{....... (7)}$$

➔ $$P_t = \frac{60000(Kw)}{\pi\ PCD\ Z} \qquad \text{....... (8)}$$

➔ $$V = \frac{\pi\ PCD\ n}{60000} \qquad \text{...... (9)}$$

➔ $$K_v = \frac{6.1+V}{6.1} \quad \text{(cut, milled profile) [2]} \qquad \text{...... (10)}$$

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Using equation (3), (4), (8), (9), (10), we get:

➔ $$\sigma = \frac{\left(\frac{6.1+V}{6.1}\right)\left(\frac{60000\ (Kw)}{\pi\ m\ Z^2}\right)}{C_1\ m^2\ Y} \qquad \text{........ (11)}$$

After substituting eq (9) in eq (11), we get:

➔ $$\sigma = \frac{6.1[\,60000\,(Kw)] + \pi\, m\, Zm\,(Kw)}{6.1\,\pi\, Z^2\, C_1\, m^3\, Y}$$ ……… (12)

In order to simplify the calculations in this Study, let us consider the following:

➔ A = 6.1 [60000 (Kw)]

➔ B = π Z^2 (Kw)

➔ C = 6.1 π Z^2 C_1 Y

Thus, the final equation is:

➔ $$\sigma = \frac{C + (B\, m^2)}{A\, m^3}$$ ………… (13)

As we seek module (m) as a function of Strength (S), we must solve by Substituting the eq (13) in eq (6). Considering the maximum condition, we solve by using **σ =S.** Due to the complexity of the equation computer aided mathematic tool could be helpful [10]. Since we have here the equation of order 3 in m, the three solutions are given below (where x=m):

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Solve[SA (x^3) - B (x^2) - C == 0, x]
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Out[11]=

$$\left\{\left\{x \to \frac{1}{3}\left(\frac{B}{SA} + \frac{2^{1/3} B^2}{SA\left(2B^3 + 27\,C\,SA^2 + 3\sqrt{3}\sqrt{4B^3\,C\,SA^2 + 27\,C^2\,SA^4}\right)^{1/3}} + \frac{\left(2B^3 + 27\,C\,SA^2 + 3\sqrt{3}\sqrt{4B^3\,C\,SA^2 + 27\,C^2\,SA^4}\right)^{1/3}}{2^{1/3}\,SA}\right)\right\},\right.$$

$$\left\{x \to \frac{B}{3\,SA} - \frac{(1 + i\sqrt{3})\,B^2}{3\,2^{2/3}\,SA\left(2B^3 + 27\,C\,SA^2 + 3\sqrt{3}\sqrt{4B^3\,C\,SA^2 + 27\,C^2\,SA^4}\right)^{1/3}} - \frac{(1 - i\sqrt{3})\left(2B^3 + 27\,C\,SA^2 + 3\sqrt{3}\sqrt{4B^3\,C\,SA^2 + 27\,C^2\,SA^4}\right)^{1/3}}{6\,2^{1/3}\,SA}\right\},$$

$$\left.\left\{x \to \frac{B}{3\,SA} - \frac{(1 - i\sqrt{3})\,B^2}{3\,2^{2/3}\,SA\left(2B^3 + 27\,C\,SA^2 + 3\sqrt{3}\sqrt{4B^3\,C\,SA^2 + 27\,C^2\,SA^4}\right)^{1/3}} - \frac{(1 + i\sqrt{3})\left(2B^3 + 27\,C\,SA^2 + 3\sqrt{3}\sqrt{4B^3\,C\,SA^2 + 27\,C^2\,SA^4}\right)^{1/3}}{6\,2^{1/3}\,SA}\right\}\right\}$$

Figure 4.0: Mathematical Computation of the equation **σ =S**

Since two of the solutions of the module are imaginary, we have to consider only the real solution of the module. After Substituting the real solution of m in eq (5), it was primarily found that the module is proportional to the S^{-1}. Therefore, the Performance of the Structural element i.e. Spur Gear is given by:

$$\textbf{Performance (P) = Mass (M)} \propto \left(\frac{\pi Z^2}{4}\right) C_1 \left(\frac{\rho}{S}\right)$$ …… (14)

From the above equation it is clear that the **Material Index of Spur Gear is** $\left(\frac{S}{\rho}\right)$**, Maximizing the Material Index will give as Minimized Mass** and hence our Objective is Fulfilled. When the performance equation is investigated, it is seen that there are two important material properties that minimize the mass. One of them is fatigue strength of the gear material that is directly related to ultimate tensile strength (Sut) of the material and the other is density of the gear material. Therefore, strength must be high while density is low to minimize the mass of a spur gear. That is to say S must be high to select optimum gear material based on the bending fatigue failure.

2.3 APPLICATION OF ASHBY CHART

There are a lot of materials, and each has a lot of properties. We need a good way to display and compare them. A useful method of doing this is by plotting them as Material Property Charts, sometimes called 'bubble' or 'Ashby' charts, with one property on one axis and another property on the other. Each material has a range of values for each property, depending on the exact composition, grade, heat treatment, supplier etc. The materials are represented on the chart as ellipses or 'bubbles', whose width and height are determined by the range of the value of the properties [7]. Note: that the data of class of material obtained from the chart are approximate, because there could many variations within a particular class of materials. The Charts include Properties like Strength, Youngs Modules, Density, Thermal Expansion, Thermal Conductivity, Embodied energy per cubic metre, Relative cost per unit Volume etc. These charts can be used in many different ways. In GRANTA EduPack they are interactive and enable you to find out more about a property or material.
In this study we will be looking at the Strength Vs. Density Chart, Given Below.

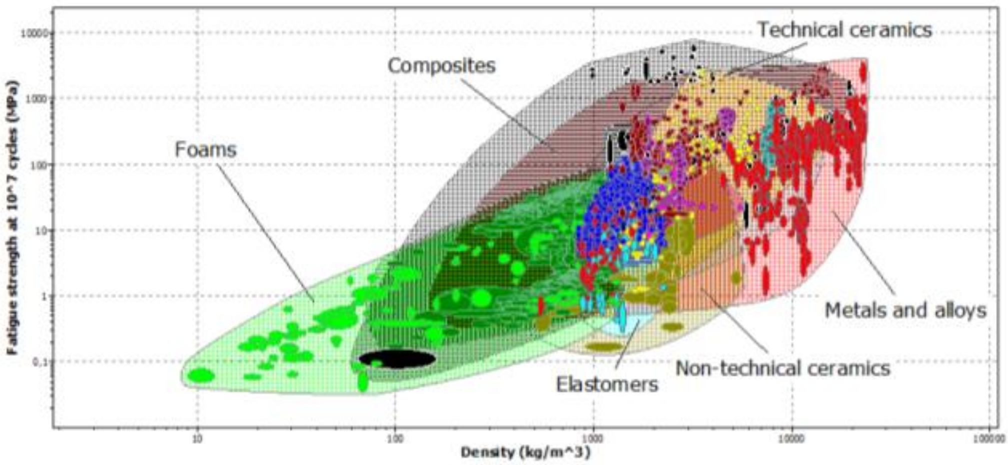

Figure 5.0: Material Families in Fatigue Strength Vs. Density.

2.4 SCREENING

The Following Steps Should be meticulously executed to perform the Screening Process.

- We have MI (Material Index) = $\left(\frac{S}{\rho}\right)$, in order increase MI we have to increase the Strength. Hence, we have to select the Selection guideline represented by $\left(\frac{\sigma}{\rho}\right)$, we have designated **S** as Strength instead of **σ** and we will continue to do so in order to avoid confusion (Selection guideline is shown in the following graph).

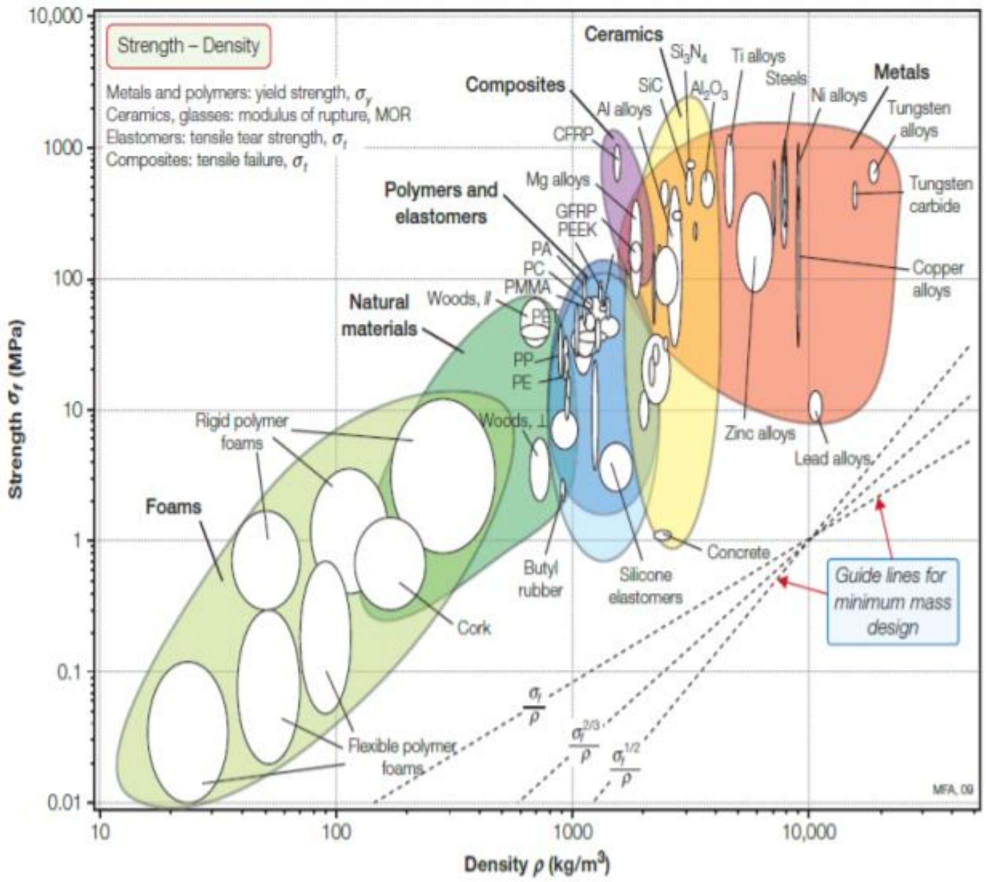

Figure 6.0: Strength Vs. Density Graph showing Selectin Guidelines.

- Translate the selected guideline parallel to itself such that we get maximum number of materials above it (shown in figure 7). The Materials which lies higher on the Guideline have high Strength and high Density and Vice Versa. The slope line separates the eliminated materials (grey locations). And the materials above the line can be a candidate considering the material index. In this study, some of the candidate materials for spur gear design were labelled as seen in Figure 7
- Select the Material which best fits the requirements. In this case the metals class of materials is chosen because as it is also reported in the literature [1], the gear materials are commonly selected from metals for industrial engineering applications. (In figure 7 Strength Vs Density graph of only metals is shown).

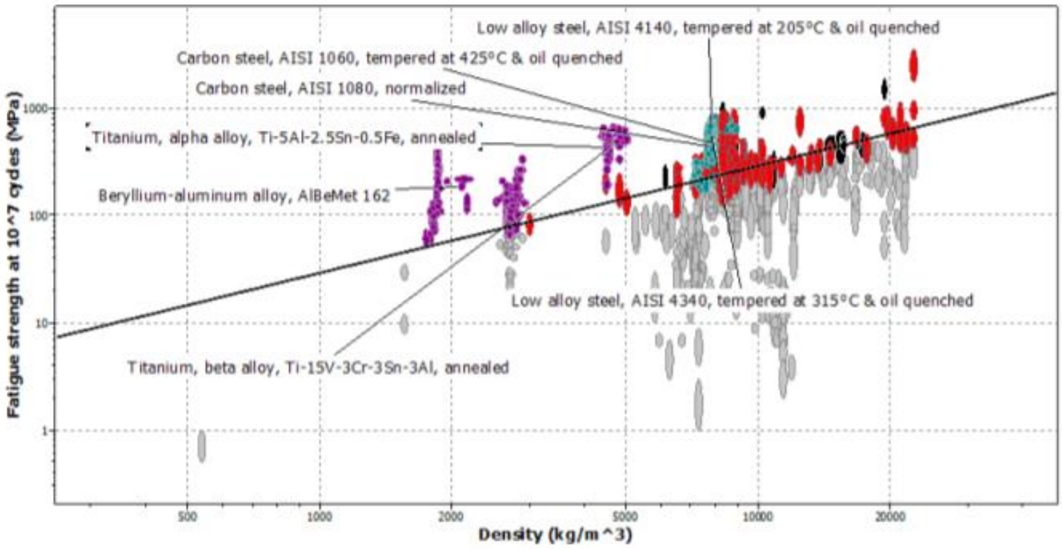

Figure 7.0: Strength Vs. Density Graph only for metals

The material index values according to the selected candidate materials were ranked and given in Table I. The properties of the selected candidate materials were listed in Table II.

No.	Material	Material Index Value
1	Titanium, beta alloy, Ti-15V-3Cr-3Sn-3Al, annealed	0.105
2	Titanium, alpha alloy, Ti-5Al-2.5Sn-0.5Fe, annealed	0.0958
3	Beryllium-aluminium alloy, AlBeMet 162	0.0901
4	AISI 4140 tempered @205C & oil quenched	0.0811
5	AISI 4340 tempered @315C & oil quenched	0.0808
6	AISI 4140, tempered at 315°C & oil quenched	0.0738
7	AISI 4340, tempered at 425°C & oil quenched	0.0711
8	AISI 1080 tempered @205C & oil quenched	0.0656
9	AISI 1080 tempered @425C & oil quenched	0.0649
10	AISI 1060, tempered at 425°C & oil quenched	0.0572
11	AISI 4140, normalized	0.0551
12	AISI 1080, normalized	0.0547

Table-1: Material Index Values

No.	Density, kg/m3	Young Modulus, GPa	Yield Strength, MPa	Tensile Strength, MPa	Hardness, HV	Poisson's ratio
1	4750	108	749	770	225	0.35
2	4460	107	758	793	344	0.32
3	2070	179	193	262	115	0.165
4	7800	208	1480	1600	455	0.285
5	7800	205	1430	1550	435	0.285
6	7800	208	1290	1400	400	0.285
7	7800	205	1230	1320	385	0.285
8	7800	200	880	1180	360	0.285
9	7800	200	855	1160	350	0.285
10	7800	208	685	965	280	0.285
11	7800	208	595	915	275	0.285
12	7800	200	470	905	270	0.285

Table-2: Material Properties

2.5 RANKING

The above given table gives us a list of Potential Candidate materials. Above given materials do fulfill the required Design Requirements and also obey the constraint of Bending failure. In the process of selecting the optimum material from the narrowed down list, we shall also consider the Relative cost per unit volume and the manufacturability. The Chart below portrays the plot of the Relative cost per unit volume and Strength. After translating the guideline of the MI = $\left(\frac{S}{\rho}\right)$, it can be easily deduced that the Ti alloys are on the topmost part of the Line and hence is too expensive to be use in the manufacturing of the gear.

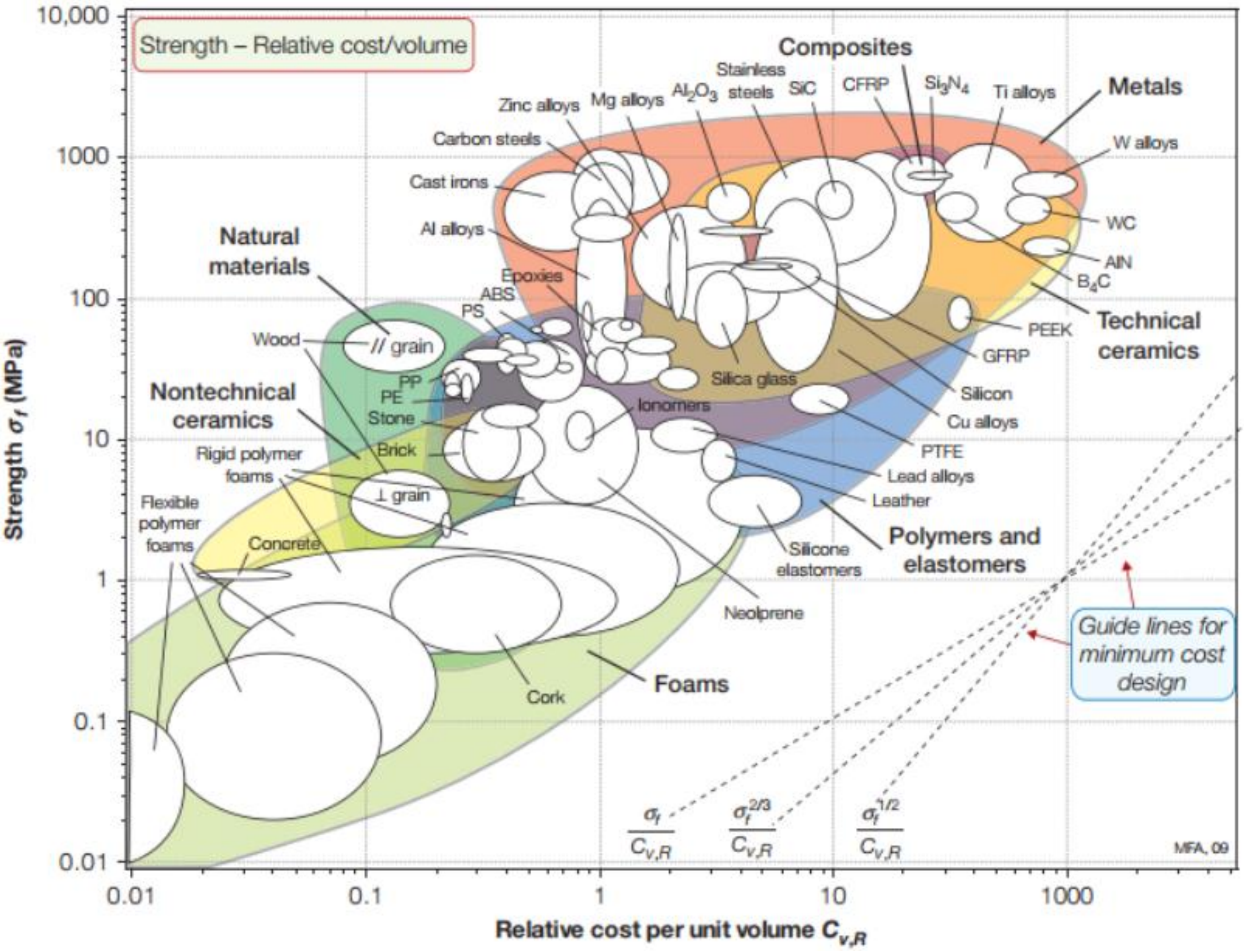

Figure 8.0: Relative cost Graph

The lightest steel was determined as AISI 4140 tempered at 205°C and oil quenched. However, the heaviest recommended material even was found approximately 15% lighter than the lightest steel. And the weight reduction for other recommended materials that are titanium alpha alloy and beryllium-aluminum alloy compared to the lightest steel was approximately 30% and 51%, respectively. When the heaviest steel (AISI 1080, normalized,) was considered, the weight gain can be 63% [3]. Such approach can give newer ideas for material application and makes the job easy (if the cost criteria is of not much importance materials such as Ti alloys and BeAlMet).

2.5 DOCUMENTATION

Gears are usually made up of Iron, Steel, Bronze and Phenolic resins. Gray cast iron gears are very cheap, excellent machineability, generate less noise and have a very good wear Strength. They are not preferred for long term use due to fact that they have very low Strength. Bronze are not much popular in Spur gear design but are indeed extensively used in worm gear work because they can withstand high sliding velocity. It can be easily worked, has low coefficient of friction and also can be used where corrosion is a major problem. There are two kinds of steels, one is called alloy steels and the other plain carbon steel. The plain carbon alloy usually has some alloy content such as Mg and Si while alloy steels contain Ni, Cr, Mo and also sometimes Mg, Si as well. The Alloy steels are Stronger and have more Fatigue Strength than that of plain carbon steels. Although expensive Steels gears have higher load carrying capacity. The Non-Metallic gears are generally used when load is light and pitch line velocity is low. Usually pinions are made up of phenolic resins such as Bakelite or Celeron while the counterpart is made up of cast iron. They can tolerate errors in tooth profile. They have low Young's Modulus and are likely to be affected by water.

3.0 CONCLUSION

This Technique of Material Selection using Ashby Charts have proven to be of great use for Optimizing the Structural element and thus increasing its performance. This method is extremely accurate and easy to comprehend and has shown a drastic improvement in the material selection process. For instance, new generation materials such as titanium alloys or beryllium alloys provided approximately 36% to 63% weight reduction compared to conventional gear materials. The charts summarize material properties in a compact, easily accessible way, showing the range spanned by each material family and class. By choosing the axes in a sensible way, more information can be displayed. Thus, in this study, the Ashby method was used to investigate lightweight, high strength materials for spur gear design. Material index values of the materials based on the bending fatigue strength were determined and ranked. The ranking was also in a good agreement with spur gear design results.

VARIABLE DEFINATIONS

- Z = number of teeth
- PCD = pitch diameter,
- m = module,
- S = Fatigue strength of gear material,
- ρ = density, kg/m3
- F = face width, mm
- P_t = Transmitted load, kN
- n = speed, rev/min
- Kw = power (in kW)
- Kv = dynamic factor
- Y = Lewis form factor
- σ = bending stress, MPa
- C_1, A, B, C = coefficients
- V = pitch-line velocity, m/s

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