A STUDY OF FUZZY SUBGROUPS AND CHARACTERISTICS OF FUZZY SUBGROUPS

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Abstract - *Fuzzy group theory has many applications in Physics, chemistry, bio informatics and computer science problems. Applications of fuzzy set theory have also been established in many areas as engineering, medicine, psychology, economics, robotics, etc In this paper the preliminaries required for the concept of Fuzzy subsets, Fuzzy subgroups and properties of fuzzy groups, introduction to the concept of normal fuzzy groups were investigated.*

Key Words: *Membership function, Fuzzy sets, Fuzzy Subgroup, Normal fuzzy subgroup*

1. INTRODUCTION

The Mathematical model of Fuzzy the concept was introduced by Lotfi. A.Zadeh(Electrical Engineering and Electronics Research Laboratory University of California Berkeley). He was Benjamin Franklin Medal Winner in Electrical Engineering 2009. He is known as the Father of Fuzzy Logic. In 1965 he laid the foundation of a comprehensive and mathematically sound logic of imprecisely or ambiguously defined sets known as "Fuzzy SETS. The Fuzzy theory has grown into a many faceted field of inquiry and contributing to a wide spectrum of areas ranging from para mathematics to human perception and judgment. Fuzzy sets have been suggested for handling the imprecise real world problems by using truth values ranging between the usual" True" and "False".

For example, consider the collection of all small natural numbers or that of all the students with brilliant academic career or the collection of all tall men etc. All the collections defined above do not constitute sets in the usual mathematical sense. Yet, the fact remains that such imprecisely defined"collections" play an important role in human thinking particularly in the areas of pattern recognition, communication of information etc.

2. PRELIMINARIES

A. Membership Function

In the classical set theory, the characteristic function of the crisp set assigns a value of either '1' or '0' to each individual in the universal set, thereby discriminating between members and non-members of the crisp set. This function can be generalized such that the values assigned to the elements of the universal set fall within a specified range and indicate the membership grade of these elements. Such a function is called the Membership function.Larger values denote higher degrees of set membership. The range of values of membership function is the unit interval [0, 1].

Two distinct notations of membership function of a Fuzzy set A

i. $\mu_A : X \to [0, 1]$
ii. $A : X \to [0, 1]$

B. Fuzzy set

A Fuzzy set A in X is characterized by a membership function $\mu_A(x)$ which associates with each point in X a real number in the interval [0, 1] with the value of $\mu_A(x)$ at X representing the grade of membership of X in A. The set A in X is called the Fuzzy set.

- $A=\{(x,\mu_A(x))|x\in X\}$
- The maximal membership of x in A is '1'
- The minimal membership of x in A is '0'
- A Fuzzy set is an extension of a classical set whereas classical set is a special case of a Fuzzy set.

EXAMPLE

Let X be the real line R^1 and let A be the Fuzzy set of numbers which are much greater than 1. Then

$\mu A(0) = 0$

$\mu A(5) = 0.01$

$\mu A(10) = 0.2$

$\mu A(100 = 0.95$

$\mu A(500) = 1$

C. Representation of Fuzzy sets

A Fuzzy set can be represented in following ways:

Let X be a classical set with n elements. ie $X = \{x_1, x_2, ...x_n\}$.Then the Fuzzy set A can be expressed in one of the following three ways:

1. $A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + ... + \mu_A(x_n)/x_n$
2. $A = \{(x_1, \mu_A(x_1))................\}$
3. $A = \{\mu_A(x_1), ...\mu_A(x_n)\}$

If X is continuous then Fuzzy set A is expressed as

$A = \int \mu_A(x)/x \, dx$

D. Fuzzy variable

Several Fuzzy sets representing linguistic concepts such as low, medium, high and so on are often employed to define states of variable. Such a variable is usually called a Fuzzy variable. For example, temperature within a range [$T1$, $T2$] is characterized as a Fuzzy variable.

E. Types of Fuzzy Sets

i. Interval-valued Fuzzy sets

The interval valued Fuzzy sets are defined by the functions of the form $\mu : X \to \varepsilon([0, 1])$ where $\varepsilon([0, 1])$ denotes the family of all closed intervals of real numbers in [0,1].

$\varepsilon([0, 1]) \subseteq P([0, 1])$.

ii. L-Fuzzy sets

Let X be a set and L be a lattice. An L-Fuzzy set of X is characterized by a mapping $\mu : X \to L$

iii. Level-n Fuzzy sets

Level -n Fuzzy sets are defined by the functions of the form $\mu : \mathcal{F}(X) \to [0, 1]$ where $\mathcal{F}(X)$ denotes the Fuzzy power set of X. Level-2 Fuzzy sets can be generalized into Level-3 Fuzzy sets by using a universal set whose elements are Level-2 Fuzzy sets. Higher level Fuzzy sets can be obtained recursively in the same way. Their membership Functions have the form $\mu : \mathcal{F}(X) \to \mathcal{F}([0, 1])$.

F. Operations of Fuzzy Sets

i. Standard Fuzzy Complement

Given a Fuzzy set A defined on a universal set X, its compliment $\bar{A}$ is another Fuzzy set on X by $\bar{A}(x) = 1 - A(x)$ ∀ $x \in X$.

ii. Standard Fuzzy Union

Consider a universal set X and two Fuzzy sets A and B, defined on X.

$(A \cup B)(x) = max\{A(x), B(x)\}$ ∀ $x \in X$

iii. Standard Fuzzy Intersection

$(A \cap B)(x) = min\{A(x), B(x)\}$ ∀ $x \in X$

G. α - Cuts and Strong α -CUTS

Given a Fuzzy set A defined on X and any number α [0, 1] , the α *cut*, $\alpha \in A$ and the strong α *cut*, $^{+\alpha}A$ are the crisp sets

$^{\alpha}A = \{x / A(x) \geq \alpha\}$

$^{+\alpha}A = \{x / A(x) > \alpha$

H. Level Set

$\Lambda(A) = \{ \alpha \,/\, A(x) = \alpha \text{ for some } x \in X\}$ where Λ denotes the level set of Fuzzy set A denoted on X.

I. The Support of A

Support of A $= \{x \in X / A(x) > 0\}$

J. The Core of a Fuzzy Set

Core $(\tilde{A}) = C(\tilde{A}) = \{x \in X / \mu_{\tilde{A}}(x) = 1\}$

K. The Height of a Fuzzy Set A

$h(A) = \sup A(x)$.

A Fuzzy set A is called **normal** when $h(A) = 1$.

A Fuzzy set A is called **subnormal** when $h(A) < 1$

3. FUZZY SUBSETS AND FUZZY GROUPS

A. Fuzzy Subsets

A Fuzzy subset of X is a function from X into [0, 1]. The set of all Fuzzy subsets of X is called the Fuzzy power set of X and is denoted by $FP(X)$

Let $\mu \in FP(X)$. Then the set $\{\mu(x) \,/\, x \in X\}$ is called the image of μ and is denoted by $\mu(X)$ *or* $Im(\mu)$. The set $\{x / x \in X, \mu(x) > 0\}$, is called the support of μ and is denoted by μ^*

If $\mu \in FP(X)$ then

Support of $\mu = \{x \in X / \mu(x) > 0\}$

Height of $\mu = \sup_{x \in X} \mu(x)$

The cross over point of $\mu = \{ x \,/\, \mu(x) = \frac{1}{2} \}$.

μ is said to be normal if height of $\mu = 1$; otherwise μ is called a subnormal Fuzzy subset.

B. Fuzzy Subsets and Operations

Let $Y \in X$ and $a \in [0, 1]$. Define $a_Y \in FP(X)$ as follows:

$$a_Y(x) = \begin{cases} a \text{ for } x \in Y \\ 0 \quad x \in X \mid Y \end{cases}$$

If Y is a singleton, say $\{y\}$ then $a_{\{y\}}$ is called a Fuzzy point (or Fuzzy singleton), and is sometimes denoted by y_a. Let 1_Y denote the characteristic function of Y. If S is a set of Fuzzy singletons, then we let $foot(S) = \{y \in X \mid y_a \in S\}$

Let μ, $\nu \in FP(X)$. If $\mu(x) \leq \nu(x)$ ∀$x \in X$, then μ is said to be contained in ν (or ν *contains* μ), and we write $\mu \in \nu$ (or $\nu \in \mu$).

If $\mu \in \nu$ and $\mu = \nu$ then μ is said to be properly contained in ν (or ν properly contains μ) and we write $\mu \in \nu$ (or $\nu \in \mu$).

The inclusion relation ⊆ is a partial ordering on $FP(X)$.

$$(\mu \cup \nu)(x) = \mu(x) \vee \nu(x).$$

$$(\mu \cap \nu)(x) = \mu(x) \wedge \nu(x). \ \forall x \in X$$

For any collection, $\{\mu i \mid i \in I\}$ of Fuzzy subsets of X, where I is a nonempty index set, the least upper bound $\cup_{i \in I}\mu_i$ and the greatest lower bound $\cap_{i \in I}\mu_i$ of the $\mu_i'S$ are given by ∀ $x \in X$

$$(\cup_{i \in I}\mu_i)(x) = \vee_{i \in I}\mu_i(x)$$

$(\cap_{i \in I}\mu_i)(x) = \wedge_{i \in I}\mu i(x)$ respectively.

$$\bigcap_{i=1}^{n}\mu_i(x) = \mu_1 \cap \mu_2 \cap \ldots\ldots\ldots \cap \mu_n$$

$$\bigcup_{i=1}^{n}\mu_i(x) = \mu_1 \cup \mu_2 \cup \ldots\ldots\ldots \cup \mu_n$$

i = 1, 2, ..., n.

Let $\mu \in FP(X)$. for $a \in [0, 1]$ μa is defined as follows :
$\mu_a = \{x \in X, \mu(x) \geq a\}$ Then μ_a is called the α *cut* of μ.

RESULTS

For all $\mu, \nu \in FP(X)$

1. $\mu \subseteq \nu$, $a \in [0, 1] \Rightarrow \mu_a \subseteq \nu_a$,
2. $a \leq b$, $a, b \in [0, 1] \Rightarrow \mu_b \subseteq \mu_a$,
3. $\mu = \nu \Leftrightarrow \mu_a = \nu_a$, $\forall a \in [0,1]$

THEOREM

Suppose that $\{\mu_i \mid i \in I\} \subseteq FP(X)$.Then for any $a \in [0,1]$,

1. $(\bigcup_{i \in I}\mu_i)_a \subseteq \bigcup_{i \in I}(\mu_i)_a$
2. $(\bigcap_{i \in I}(\mu_i)_a = (\bigcap_{i \in I}\mu_i)_a$

THEOREM

L $\mu \in FP(X)$ and $\{a_i \mid i \in I\}$ be a nonempty subset of [0, 1]. Let $b = \wedge_{i \in I} a_i$ and $c = \vee_{i \in I} a_i$ Then the following assertions hold:

1. $\bigcup_{i \in I}\mu_{a_i} \subseteq \mu_b$
2. $\bigcap_{i \in I}\mu_{a_i} = \mu_c$

Let I be a non empty index set and let $\{X_i \mid i \in I\}$ be a collection of nonempty sets.
Let X denote the Cartesian product of the $X_i^j s$ namely

$$X = \prod_{i \in I} Xi = \{(x_i)_i \in I \mid (xi \in X_i, i \in I\}$$

Let $\mu_i \in FP(X_i)$, for all $i \in I$ .Define the fuzzy subset μ of X by
$\mu(x) = \wedge_{i \in I}\mu_i(x_i)\ \forall x = (x_i)_i I \in X$. Then μ is called the complete direct product of the μ_i's and is denoted by

$$\mu = \tilde{\prod_{i \in I}} \mu_i$$

C. First Decomposition Theorem

For every $A \in FP(X)$, Let A = $\bigcup_{\alpha \in [0,1]} \alpha^A$ where α^A is defined by $\alpha^A(x) = \alpha.^a A(X)$ and ∪ denotes the standard Fuzzy union.

EXAMPLE

Consider a Fuzzy set A with the following membership function of triangular shape

$$A(x) = \begin{cases} x - 1 \text{ if } x \in [2,3] \\ 3 - x \text{ if } x \in [2,3] \\ 0 \text{ otherwise} \end{cases}$$

For each $\alpha \in [0,1]$, the α - cut of A is in this case the closed interval $\alpha_A = \{x \in X \mid A(x) \geq \alpha\} = \{\alpha + 1, 3 - \alpha\}$ and the special Fuzzy set α^A employed is defined by the membership function.

$$\alpha_A = \begin{cases} \alpha \text{ if } x \in [\alpha + 1, 3 - \alpha] \\ 0 \text{ otherwise} \end{cases}$$

D. Extension Principle

Let F be a function from X into Y, and let $\mu \in P(X)$ and $\upsilon \in P(Y)$
Define the Fuzzy subsets
$F(\mu) \in FP(Y)$ and $F^{-1}(\upsilon) \in FP(X)$ by $\forall y \in Y$

$$F(\mu)(y) = \begin{cases} v \ \{\mu(x) \mid x \in X, f(x) = y\} \text{ if } f^{-1} \\ 0 \text{ otherwise} \end{cases} \text{ and}$$

$\forall x \in X$, $F^{-1}(\upsilon)(x) = \upsilon(F(x))$ Then $F(\mu)$ is called the image of μ under F and $F^{-1}(\nu)$ is called the preimage of ν under Ƒ. The least upper bound of the empty set is the zero elements.

THEOREM

Let F be a function from X into Y and g a function from Y into Z. Then the following assertions hold.

1. Forall
   $\mu 1 \subseteq \mu 2 \Rightarrow f(\mu_1) \subseteq f(\mu_2)$
   $\mu_i \in FP(X_i)$, for all $i \in I$
   $f(\bigcup_{i\in I}\mu_i) = \bigcup_{i\in I} f(\mu_i)$ andso $\forall \mu_1, \mu_2 \in fP(X)$
2. For all $\upsilon_j \in fP(Y), j \in J$, where J is a nonempty index set,

$$F^{-1}(\bigcup_{j\in J}\upsilon_j) = (\bigcup_{j\in J}F^{-1}(\upsilon_j)$$
$$F^{-1}(\bigcap_{j\in J}\upsilon_j) = \bigcap_{j\in J}F^{-1}(\upsilon_j) \text{ and}$$

Therefore

$$\upsilon_1 \subseteq \upsilon_2 \Rightarrow F^{-1}(\upsilon_1) \subseteq F^{-1}(\upsilon_2)\ \forall \upsilon_1, \upsilon_2 \in FP(Y)$$

3. $f^{-1}(f(\mu)) \supseteq \mu \in FP(X)$. *In particular, if f is an injection, then* $f^{-1}(f(\mu)) = \mu\ \forall \mu \in FP(X)$
   $f^{-1}(f(\mu)) = \mu$? μ ?ƑP(X) This means μ ↦ f(μ) is an injection from ƑP(X) into ƑP(Y) and ν ↦ $f^{1}(\nu)$ is a surjection from ƑP(Y) onto ƑP(X)
4. $f(f^{-1}(\upsilon)) \subseteq \upsilon, \upsilon \in FP(Y)$. *In particular ,if f is a surjection ,then* $f(f^{-1}(\upsilon)) = \upsilon, \forall \upsilon \in FP(Y)$
   and therefore μ ↦ f(μ) is a surjection from ƑP(X) onto ƑP(Y) and ν ↦ $f^{1}(\nu)$ is an injection from ƑP(Y) into ƑP(X)
5. $f(\mu) \subseteq \upsilon \rightarrow \mu \subseteq f^{-1}(\upsilon)\ \forall \mu \in FP(X) \text{ and } \forall \upsilon \in FP(Y)$
6. $g(f(\mu)) = (gof)(\mu)\ \forall \mu \in g(f(\mu)) = (gof)(\mu)\ \forall \mu \in FP(X) \text{ and } f^{-1}(g^{-1}(\xi)) = (gof)^{-1}(\xi)$
   $\forall \xi \in FP(Z)$

EXAMPLE

Let $X = \{x1, x2, x3, x4\}$ *and* $Y = \{y1, y2, y3, y4\}$ *and f: ƑP(X) → ƑP(Y) be defined by* $f(x_1) = f(x_2) = y_1, f(x_3) = y_3, f(x_4) = y_4$, *ν = {(y₁, .2), (y₂, .4), (y₃, .5), (y₄, 1)} Then*

$$f^{-1}(\upsilon)(x_1) = \upsilon(f(x_1) = \upsilon(y_1) = .2$$

$$f^{-1}(\upsilon)(x_2) = \upsilon(f(x_2) = \upsilon(y_2) = .2$$
$$f^{-1}(\upsilon)(x_3) = \upsilon(f(x_3) = \upsilon(y_3) = .5$$
$$f^{-1}(\upsilon)(x_4) = \upsilon(f(x_4) = \upsilon(y_4) = 1$$
$$f^{-1}(\upsilon) = \{(x_1,.2),(x_2,.2),(x_3,.5),(x_4,1)\}$$

Let $\mu = \{(x_1,.1),(x_2,0),(x_3,.5),(x_4,.9)\}$

$$f(\mu)(x) = \vee\{\mu(x) \mid x \in X, f(x) = y\} \text{iff } f^{-1}(y) \neq 0$$

$$f(\mu)(x_1) = 0.1$$
$$f(\mu)(x_2) = 0$$
$$f(\mu)(x_3) = 0.5$$
$$f(\mu)(x_4) = 0.9$$
$$f(\mu) = \{(y_1,.1),(y_2,0),(y_3,.5),(y_4,.9)\}$$

4. FUZZY SUBGROUPS

Define the binary operation "o" on $fP(G)$ and the unay operation $^{-1}$ on FP(G) as follows:

$\forall \mu, \upsilon \in FP(G)$ and $\forall x \in G$

$$(\mu o \upsilon)(x) = \vee\{\mu(y) \wedge \upsilon(z) \mid y, z \in G, yz = x\}$$

and $\mu^{-1}(x) = \mu(x^{-1})\, \mu o \upsilon$

the product of μ and υ and

μ^{-1} is the inverse of μ

THEOREM

Let $\mu, \upsilon, \mu_i \in FP(G), i \in I$.

Let $a = \vee\{\mu(x) \mid x \in G\}$

Then the following assertions hold:

$$1. (\mu o \upsilon)(x) = \vee_{y\in G}\{\mu(y) \wedge \upsilon(y^{-1}x)\}$$
$$= \vee_{y\in G}\{\mu(y^{-1}x) \wedge \upsilon(y)\} \forall x \in G$$
$$1. (a_y o \mu)(x) = \mu(y^{-1}x)\ \forall\ x, y \in G$$
$$3. (\mu o a_y)(x) = \mu(xy^{-1}) \forall x, y \in G$$
$$4. (\mu^{-1}) \Leftrightarrow 1 = \mu$$
$$5. \mu \subseteq \mu - 1$$
$$\Leftrightarrow \mu^{-1} \subseteq \mu$$
$$\Leftrightarrow \mu = \mu^{-1}$$
$$\Leftrightarrow \mu(x) \leq \mu(x^{-1})\ \forall x \in G$$
$$\Leftrightarrow \mu(x^{-1}) \leq \mu(x)\ \forall x \in G$$
$$\Leftrightarrow \mu(x)\mu = \mu(x^{-1})\ \forall x \in G$$
$$6. \mu \subseteq \upsilon \Leftrightarrow \mu^{-1} \subseteq \upsilon^{-1}$$
$$7. (\bigcup_{i\in I}\mu_i)^{-1} = \bigcup_{i\in I}(\mu_i^{-1})$$
$$8. (\bigcap_{i\in I}\mu_i)^{-1} = \bigcap_{i\in I}\mu_i^{-1}$$
$$9. (\mu o \upsilon)^{-1} = \upsilon^{-1} o\, \mu^{-1}$$

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A. Fuzzy Subgroup of G

Let $\mu \in FP(G)$. Then μ is called a fuzzy subgroup of G if

1. $\mu(xy) = \mu(x) \wedge \mu(y), \forall x, y \in G$ and
2. $\mu(x^{-1}) = \mu(x), \forall x \in G$

denoted by F(G), the set of all fuzzy groups of G .

If $\mu \in F(G)$. Let $\mu_* = \{x \in G \mid \mu(x) = \mu(e)\}$,

Where μ_* denotes support of μ.

EXAMPLE

Consider the group G = $\{1, -1, i, -i\}$ under the usual multiplication

Define μ on G by

$$\mu(x) = \begin{cases} 1 \text{ if } x = 1 \\ \frac{1}{2} \text{ if } x = -1 \\ \frac{1}{3} \text{ if } x = i \text{ or } x = -i \end{cases}$$

EXAMPLE

Let $G = (R,+)$. Define μ on G

$$\mu(x) = \begin{cases} 1 \text{ if } x = 0 \\ 0.7 \text{ if } x \in Q - \{0\} \\ 0.3 \text{ if } x \in R - Q \end{cases}$$

Then μ is a fuzzy group

EXAMPLE

Let G = (Z,+) and H = {1,-1 }, is a subgroup under

multiplication. Define $f : G \to H$ by

$$f(x) = \begin{cases} 1 \text{ when } x \text{ is even} \\ -1 \text{ when } x \text{ is odd} \end{cases}$$

Then f is a subgroup homomorphism.Define μ on G by

$$\mu(x) = \begin{cases} 1 \text{ when } x \text{ is even} \\ \frac{1}{2} \text{ when } x \text{ is odd} \end{cases}$$

Then μ is a fuzzy subgroup on G. We have

$$f(\mu)(1) = \vee\{\mu(x) \mid x \in f^{-1}(1)\}$$
$$= \vee\{\mu(x) \mid f(x) = 1\} = 1$$

$$f(\mu)(-1) = \vee\{\mu(x) \mid x \in f^{-1}(-1)\}$$
$$= \vee\{\mu(x) \mid f(x) = -1\} = 1/2$$

Therefore f(μ) is a fuzzy subgroup on H.

Define ν on H by ν(1) = 1 and ν(-1) = 1/2

$$f^{-1}(\nu)(x) = \nu(f(x)) \begin{cases} 1 \text{ when } x \text{ is even} \\ \frac{1}{2} \text{ when } x \text{ is odd} \end{cases}$$

If x and y are even , then x+y is even .Then

$$f^{-1}(\upsilon)(x+y) = 1, f^{-1}(\upsilon)(x) = 1 \text{and } f^{-1}(\upsilon)(y) = 1.$$
$$f^{-1}(\upsilon)(x+y) = 1 = 1 \wedge 1 = f^{-1}(\upsilon)(x) \wedge f^{-1}(\upsilon)(y).$$

If x is even and y is odd, then x+y is odd. Then

$$f^{-1}(\upsilon)(x+y) = 1/2, f^{-1}(\upsilon)(x) = 1,$$
$$\text{and } f^{-1}(\upsilon)(y) = 1/2$$
$$f^{-1}(\upsilon)(x+y) = 1/2 = 1.\frac{1}{2} = f^{-1}(\upsilon)(x) \wedge f^{-1}(\upsilon)(y)$$

x is odd and y is odd, then x+y is even. Then

$$f^{-1}(\upsilon)(x+y) = 1, f^{-1}(\upsilon)(x) = 1/2, \text{and } f^{-1}(\upsilon)(y) = 1/2.$$
$$f^{-1}(\upsilon)(x+y) = f^{-1}(\upsilon)(x) \wedge f^{-1}(\upsilon)(y). \text{Hence} \forall x, y \in G$$
$$f^{-1}(\upsilon)(x+y) = f^{-1}(\upsilon)(x) \wedge f^{-1}(\upsilon)(y)$$

$f^{-1}(\upsilon)$ is a Fuzzy subgroup of G

LEMMA

Let $\mu \in F(G)$. Then $\forall x \in G$

1. $\mu(e) = \mu(x)$
2. $\mu(x) = \mu(x-1)$

THEOREM

If μ is a fuzzysubgroup of G and if $x, y \in G$ with $\mu(x) \neq \mu(y)$ then $\mu(xy) = \mu(x) \wedge \mu(y)$.

THEOREM

Let $\mu \in FP(G)$.
Then $\mu \in F(G)$ if and only if satisfies the following conditions :
(1) $\mu \circ \mu \subseteq \mu$;
(2) $\mu^{-1} \supseteq \mu$

THEOREM

Let $\mu, \upsilon \in F(G)$. Then $\mu \circ \upsilon \in F(G)$ if and only if $\mu \circ \upsilon = \upsilon \circ \mu$.

THEOREM

Let $\mu \in F(G)$ and H be a group.
Suppose that f is a homomorphism of G into H. Then $f(\mu) \in F(H)$.

THEOREM

Let H be a group and $v \in F(H)$. Let f be a homomorphism of G into H. Then $f^{-1}(v) \in F(G)$.

THEOREM

Let $\{\mu i \mid i \in I\} \subseteq F(G)$. Then $\bigcap_{i\in I} \mu_i \in F(G)$

5. SOME PROPERTIES OF FUZZYSUBGROUP

Let S a fuzzy subgroup of μ iff

$$\mu_{\hat{S}}(xy^{-1}) = \mu_{\hat{S}}(x) \wedge \mu_{\hat{S}}(y) \text{ for all } x, y \in \hat{S}$$

PROPOSITION

If μ is a fuzzysubgroup, then

$$G^m = \{(x, \mu_G(x)^m \mid x \in G\} \text{ is fuzzysubgroup.}$$

COROLLARY

The fuzzysubgroup G^n is a fuzzysubgroup of G^m, if $m \le n$.

PROPOSITION

If G^i and G^j are fuzzysubgroups, then
1. $G^i \cup G^j$ is also a fuzzysubgroup if $i < j$
2. $G^i \cap G^j$ is also a fuzzysubgroup if $i < j$

6. HOMOMORPHISMS OF GROUPS

Let $f : G \to G^1$ be a homomorphism of groups.
For any fuzzyset $\mu \in G^1$. Define a fuzzyset $\mu^f \in G$ by $\mu^f(x) = \mu(f(x))$ for all $x \in G$

PROPOSITION

Let G and G^1 be groups and f be a homomorphism from G onto G^1, if

1. if μ is a fuzzy group of G^1 then μ^f is a fuzzy group of G
2. if μ^f is a fuzzy group of G the μ is a fuzzy group of G^1.

7. ABELIAN FUZZY SUBGROUPS

THEOREM

Let $\mu \in FP(G)$.
Then the following assertions are equivalent :
(1) $\mu(yx) = \mu(xy)\ \forall x, y \in G$;
Then, μ is called an Abelian fuzzysubset of G.
(2) $\mu(xyx^{-1}) = \mu(y)\ \forall\ x, y \in G$
(3) $\mu(xyx^{-1}) \ge \mu(y)\ \forall\ x, y \in G$.
(4) $\mu(xyx^{-1}) \le \mu(y)\ \forall\ x, y \in G$.
(5) $\mu \circ \upsilon = \upsilon \circ \mu\ \upsilon \in FP(G)$.

8. NORMAL FUZZY SUBGROUPS

Let $\mu \in F(G)$. Then μ is called a normal fuzzysubgroup of G if it is an Abelian fuzzysubset of G.
Let N F(G) denote the set of all normal fuzzysubgroups of G.
If $\mu, \upsilon \in F(G)$ and there exits $u \in G$ such that
$\mu(x) = \upsilon(uxu^{-1}) \; \forall \; x \in G$, then μ and
υ are called conjugate fuzzysubgroups
and $\mu = \upsilon^{u}$, where $\upsilon^{n}(x) = \upsilon(uxu^{-1}) \; \forall \; x \in G$.
Also 1_G and $1_{\{e\}}$ are normal fuzzysubgroup of G.
If G is a commutative group,
every fuzzysubgroup of G is normal.
A fuzzysubgroup of G is normal if and only if $\mu = \mu^{z} \; z \in G$.

THEOREM

Suppose $\mu \in F(G)$.
Let $N(\mu) = \{ x \mid x \in G, \mu(xy) = \mu(yx), \forall y \in G \}$.
Then $N(\mu)$ is a subgroup of G.

A. Normal Fuzzy Subgroup Of The Fuzzy Subgroup ν

Let $\mu, \upsilon \in F(G)$ and $\mu \subseteq \upsilon$.
Then μ is called a normal fuzzysubgroup of the fuzzysubgroup υ, denoted by $\mu \Delta \upsilon$,
if $\mu(xyx^{-1}) \geq \mu(y) \wedge \upsilon(x) \; \forall x, y \in G$

EXAMPLE

Let $G = \{1, -1, i, -i\}$ $G1 = \{1, -1, i, -i\}$ $G2 = \{1, -1\}$ $G3 = \{1\}$
Then $G = G1 \supset G2 \supset G3 = \{1\}$.
Also $G2 \Delta G1$, $G3 \Delta G2$ and $G1|G2$ and $G2|G3$ are abelian.
G is abelian.

Define μ on G by

$$\mu(x) = \begin{cases} 1 \text{ when } x = 1 \\ \frac{1}{2} \text{ when } x = -1 \\ \frac{1}{3} \text{ when } x = i or -i \end{cases}$$

μ is a fuzzy group on G.

Now define μ_2 on G by

$$\mu_1(x) = \begin{cases} 1 \text{ when } x = 1 \\ \frac{1}{4} \text{ when } x = -1 \\ \frac{1}{5} \text{ when } x = i or -i \end{cases}$$

Then μ_1 is a fuzzy subgroup of G such that
$\mu 1 \subseteq \mu . \mu 1(xyx^{-1}) = \mu_1(y) \wedge \mu(x), \; \forall \; x, \; y \in G$
Hence $\mu_1 \Delta \mu$

THEOREM

If $\mu \in N\,F(G)$ and $\upsilon \in F(G)$,
then $\mu \cap \upsilon$ is a normal fuzzysubgroup of υ.

9. CONCLUSION

This project includes the preliminaries required for the concept of Fuzzy subsets and Fuzzy subgroups and properties of fuzzy groups, introduction to the concept of normal fuzzy groups. Fuzzy sets are sets whose elements have degree of membership. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition - an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1]. This work focused on the study of fuzzy subsets and fuzzy subgroups. This helps to investigate the concept of fuzzy groups and obtain some results. Fuzzy group theory has many applications in Physics, chemistry, bioinformatics and computer science problems. Applications of fuzzy set theory have also been established in many areas as engineering, medicine, psychology, economics, robotics, etc. Thus in this modern society the importance of fuzzy set is realized and developments on this topic are increasing day by day. A study on the structure of the collection of fuzzy subgroups will be a prosperous venture. This project has made a humble beginning in this direction.

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