TOTAL CO-INDEPENDENT DOMINATION NUMBER IN JUMP GRAPH

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1. **Introduction;** All raphs in this paper will be finite and undirected without loops and multiple edges. As usual p = |V| and q=|E| denote the number of vertices and edges of a jump graph J(G) respectively 8in general, we use X denote the sub graph number induce by the set of vertices X. N(v0and N[v] denote the open and closed neighborhood of vertex v, respectively. A set D of vertices in J(G)is a denoted set if every vertex in V –D is adjacent to some vertex in D. The domination number $\gamma\ (J(G))$ is the minimum cardinality of a dominating set of J(G), If J(G) is connected graph then a vertex cut of J(G) is a subset R of V with the property that the sub graph of J(G) induced by J(V) – R is disconnected If J(G) is not a complete graph then the vertex connectivity number k(J(G)) is the minimum cardinality of a vertex cut . I J(G) is complete graph $J(k_p)$ it is known that k(J(G)) = p-1. For terminology and notations not specifically defined hence we refer reader to [4] For more details a cut domination number and its related parameters, we refer to [5] [11] and [13]. A dominating set S of J(G) is called a connected dominating set
   if the induced sub graph S is connected. The minimum cardinality of a connected dominating set of J(G) and is denoted by γ c(JG)[12] .A dominating set S of J(G) is called non split dominating set if he induced sub graph J(V)-S is connected . The minimum cardinality of a non-split dominating set S of J(G)and I denoted by γ_{ns} [J(G)] [8]. A dominating set S of J(G)
   called total dominating set of J(G) is called total domination number of J(G) is denoted by γ_t(J(G))[5]. Many application of domination in graphs can be extended to the co-independent domination. For example the routing protocols in such networks are typically based on the con ept of a virtual backbone in ad hoc wireless network. This motivates us to introduce the concept of total co-independent domination in a jump graph.

2. **Total Co-independent Domination Number**

Definition: A total dominating set S of a graph J(G)= (V,E) is called total co-in pendent dominating set, if the induced sub graph V- S has no edge and has at least one vertex. The minimum cardinality of a co-independent dominating set of J(G) is called the total co-independent J(G) is denoted by γ_{tcoi} (J(G)) .
A Total co-independent dominating set S is said to be minimal if no proper subset of S is total co-independent dominating set.
Observation 2.1 :A non empty graph J(G) is without isolated vertices if and only if it admits a total co-independent dominating set.
Observation 2.2; A total co-independent dominating set D of a jump graph J(G) is minimal irf and only if for each vertex v∈D, one of the following condition is satisfied.
9i) There exists a vertex u∈V such that N(u) ∩D ={v}
(ii) V –(D –{}) is independent set . Therefore D-{v} is total co-independent dominating set of J(G) a contradiction. Hence one of the given conditions is satisfies .The converse is straight forward
The following observations are immediate. **Observation2.3:** For any cycle C_p, $\gamma_{t\,coi}(C_p)$= p – p_3
Observation 2.4: For any path P_p, $\gamma_{t\,coi}$ (P_p)= p – P_3
:Observation 2.5: For any wheel W_p with –p vertices $\gamma_{t\,coi}(W_p)$= 1+ p-1.2
Observation 2.6: For any complet graph K_p $\gamma_{t\,coi}(K_p)$ = p – 1.
Observation 2.7 For any complte bipartite graph $K_{r,s}$ where r≤s γ_{tcoi} $(K_{r,s})$ = r+1.

Proof:Let J(G) be a graph with p≥ 3 vertices and has isolated vertices, then
2 ≤ $\gamma_{t,coi}$ (J(G)) ≤ p -1. Further the equality of upper bound is attained if J(G) is P_3 or J(G) is two star, and the upper bound attained if J(G) is complete graph Kp or G = P_2 U P_3.

Proof: Let J(G) be graph with p≥ 3 vertices and has no isolate4d vertices and D total co-independent dominating set of J(G).Then obviously D is total dominating set Hence

$2 \leq \gamma_{t,coi}$ (J(G)). For the upper bound, suppose that D – V- {u} where u is a pendent vertex with respect o some spanning tree of G. Clearly D is total-co-independent dominating set of G. Therefore $\gamma_{t,coi}$ (J(G)) ≤ p -1.. Hence $2 \leq \gamma_{t,coi}$ (J(G)) ≤ p -1..

Proposition 2.9 : For any jump graph J(G)=(V,E) with no isolated vertices and |V|≤ 3,
(i) γ(J(G) ≤ γ_s(J(G)) ≤ $\gamma_{t\,coi}$(J(G)) (ii) γ(J(G) ≤ γ_t(J(G)) ≤ $\gamma_{t\,coi}$(J(G))

Proof: Let J(G) =(V,E) be a jump graph with no isolated vertices. Suppose that S⊆ V is any minimum total co-independent dominating set of J(G).Since for any graph J(G) any total co-independent dominating set S is also split dominating set and every dominating set is also dominating set. Hence γ(J(G) ≤ γ_s(J(G)) ≤ $\gamma_{t\,coi}$(J(G)). Similarly we can prove (ii)

Proposition 2.10: If J(G)=(V,E) is a jump geaph with no isolated vertices and |V| ≥ 3 and H is spanning sub graph with no isolated vertices and has vertices greater than

γ_t(J(G))) ≤ $\gamma_{t\,coi}$(J(G)).

Proof:Let S be any minimum totao co-independent dominating set of H Then obviously from the definition of the total co-independent domination, S is also total dominating set of H.n Therefore S is total dominating set of J(G), Hence γ_t(J(G))) ≤ $\gamma_{t\,coi}$(J(G)).

Proposition 2.11: Let J(G) be a graph with D is minimal total co-independent dominating set then V-D is independent set of J(G)..

Proof: Let D be a minimal total co-independent dominating set of J(G). Suppose that V – D is not independent dominating set of J(G). Since D is total co-independent dominating set of J(G), then there exist a vertex u such that u is not dominated by any vertex in V – D. Since J(G) has total co-independent dominating set, then J(G) has no isolated vertices, therefore u is dominated by at least one vertex in D – {u} . Thus D – {u} is total co-independent dominating set of J(G) which contradicts the minimality of D. Thus every vertex in D is adjacent with at least one vertex in V – D and V –D is independent dominating set of J(G)..

Proposition 2.12 If J(G)=(V,E) is a jump graph with noib isolated vertices and |V|≥ 3 and H is spanning dsub graph with no isolated vertices and has vertices greter than two og J(G) then $\gamma_{t\,coi\,(}$ (J(H)) ≤ $\gamma_{t\,coi\,(}$ (J(G)).

Proof: As the number of independent vertices may increase in any connected spanning sub graph J(H) of J(G) we can still maximize the set V – D which results in the decrease of the value V $\gamma_{t\,coi\,(}$ (J(G)). Hence $\gamma_{t\,coi\,(}$ (J(H)) ≤ $\gamma_{t\,coi\,(}$ (J(G))..

Observation 2.13: For any jump graph J(G) any total co-independent dominating set of J(G) contains all the support vertices..

Proof: Suppose the graph J(G) has a total co-independent dominating set D and let v be support vertices does not belongs to D then clearly the pendent vertex which adjacent to v can not belong to D from the definition of total co-independent dominating set. Hence D is not a dominating set which is contradiction.

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