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Modal Analysis of Pinned-Pinned Beam using Numerical, Analytical and Experimental Techniques

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Abstract- *In this study, Modal analysis of rectangular cross-section uniform beam is investigated numerically, analytically and experimentally under the boundary conditions (pinned-pinned) simply supported. Modal analysis is a technique for determining the system's normal modes, normal shapes and frequencies as well as for better understanding the system as a whole. In this paper transverse vibration modal analysis of simply supported wooden beam is carried out 3 by different approaches. Numerical analysis is accomplished by ANSYS based FEA software, Analytical solution is carried out by Euler-Bernoulli's beam theory under the assumption effect of rotary inertia and shear deformation of beam is neglected and experimental analysis is carried out by the universal vibration apparatus. To confirm these results numerical, analytical and experimental results together are found to be in good agreement.*

Key Words: Finite element analysis, Free vibration, Modal analysis, pinned-pinned beam.

1.INTRODUCTION

"In general, mechanical system consists of two systems, namely discrete system and continuous system. Discrete system are those systems in which mass ,elasticity and damping are concentrated at a point. These system are modelled as discrete one and the governing equation for these type of system is ordinary differential equation. On the other hand if the mass, elasticity and damping is distributed throughout the system that means each point of the continuous system has its own degree of freedom and can vibrate independently. The governing equation for continuous system is partial differential equation that depend on time as well as spatial coordinates"[1].In practical life all mechanical system consists of structural components which have mass ,damping and elasticity are continuous system examples are beams,rods,taut strings,plates and shells. These structural components have infinite number of degrees of freedom and when external frequency coincides with one of its natural frequency that will cause serious vibrations with large amplitudes which are detrimental to our structures. In this section, modal analysis of structural wooden beam is investigated. Simply supported beam is used in bridges, buildings, beds of machine tools, roofs of house's and bridge decks. So it becomes important to study its vibration.

2. NUMERICAL ANALYSIS FOR TRANSVERSE VIBRATION OF A PINNED-PINNED BEAM.

Using finite element modelling software, engineers can use ANSYS to gain a deeper understanding of a structure's dynamic properties[2].we use ANSYS structural package to analyses the free vibration of pinned-pinned beam. In today's world, the finite element method can be used to solve mathematical models of a wide range of engineering problems ranging from stress analysis of truss, beam and frame structures or complicated machines to dynamic responses of automobiles, trains or planes under various mechanical, thermal, or electromagnetic loads[5].In this section we will calculate first 3 dominating fundamental frequencies of wooden beam with pinned-pinned support using ANSYS based software.

3.1 NUMERICAL(FEA) RESULTS.

Consider a uniform beam of length L=780mm and cross section area A= 30×5mm^2 made of soft wood having young's modulus E= 9.9× $10^9 N/m^2$, density ρ = 468kg/$m^3$and poisson's ratio μ = .33 as shown below in fig.

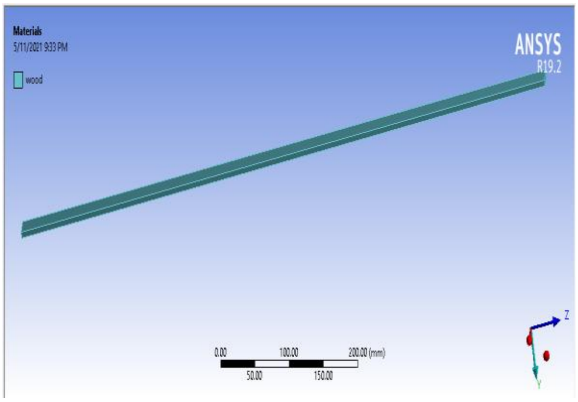

Fig-1: wooden solid beam model using ansyswork bench

The end aim of using the finite element technique is to construct a finite element mesh of the beam. The mesh of the beam can be generated directly i.e. nodes and elements can be generated one at a time [3]. Its assumed that all elements that are used to create mesh of a beam are equivalent. The numerical results were obtained using the Ansys software as seen in the table-1 below.

Table-1: Numerical results of beam using Ansys program

Mode	Numerical frequency(hz)
1	17.14
2	68.561
3	154.27

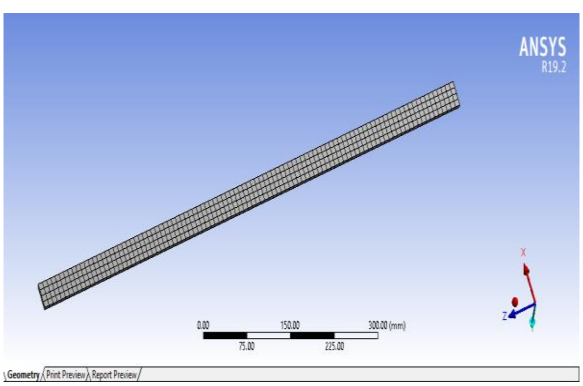

Fig-2: Finite element model of wooden beam

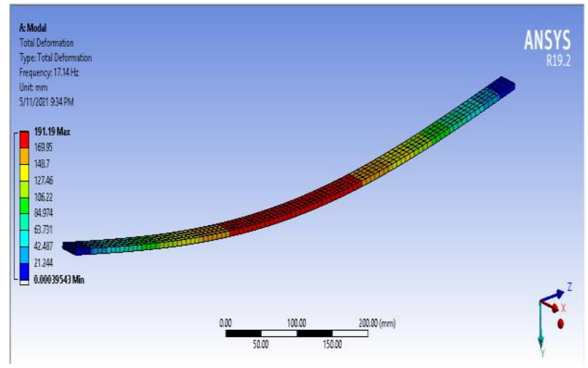

Fig-3: Ist mode shape of beam using workbench

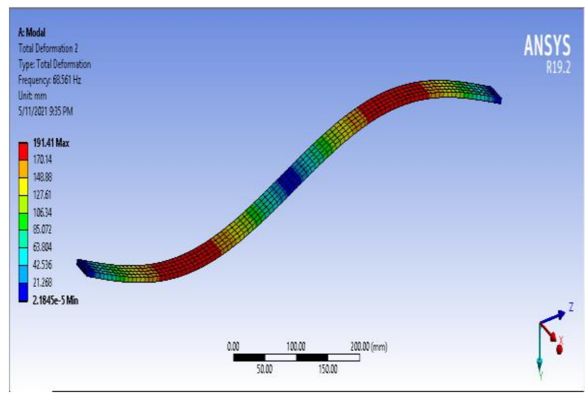

Fig-4: 2nd mode shape of beam using workbench

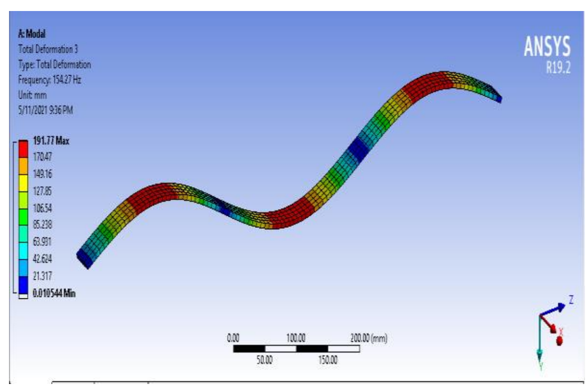

Fig-5: 3rd mode shape of beam using workbench

3. MODAL ANALYSIS FOR TRANSVERSE VIBRATION OF A PINNED-PINNED BEAM USING ANALYTICAL METHOD.

There are two different types of beam theories that are in common use and they are Euler-Bernoulli beam and Timoshenko beam theory that are used in structural analysis of beams. In the Euler-Bernoulli beam theory, effects of rotatory inertia and shear deformation is neglected [1]. The neutral axis passes through beam centroid of each planar cross section remains in perpendicular before and after the bending of beam. This theory is suitable for describing the behavior when the length of span is very larger than its thickness. On the other hand,Timoshenko beam theory, in this model planar cross section doesn't remain in perpendicular with the neutral axis after bending of beam. By considering the shear strain effects into account, this model is suitable for realistic short or thick beams. These models are valid for beams with uniform cross section. Considering an Euler-Bernoulli beam model with uniform cross section undergoing transverse vibrations. The equation of motion for the forced transverse vibration of a uniform beam is [4].

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w}{\partial x^2}(x,t)\right] + \rho A(x)\frac{\partial^2 w}{\partial t^2}(x,t) = f(x,t) \quad (1)$$

For a uniform beam

$$EI\frac{\partial^4 w}{\partial x^4}(x,t) + \rho A\frac{\partial^2 w}{\partial t^2}(x,t) = f(x,t) \quad (2)$$

For free vibration

$$c^2\frac{\partial^4 w}{\partial x^4}(x,t) + \frac{\partial^2 w}{\partial t^2}(x,t) = 0 \quad (3)$$

Where

$$c = \sqrt{\frac{EI}{\rho A}}$$

"A is the uniform beam's cross-sectional area, I is the moment of inertia, where E is the young's modulus of elasticity, w is the beam's transverse displacement, ρ is the density of beam, t is the time and x is the position along the beam from 0 to L" [4].

The variable separable method can be used to find the solution to the partial differential equation.

$$w(x,t) = W(x)T(t) \quad (4)$$

Substitute eq(4) in (3) we get

$$\frac{c^2}{W(x)}\frac{d^4W(x)}{dx^4} = -\frac{1}{T(t)}\frac{d^2T(t)}{dt^2} = a = \omega^2 \quad (5)$$

Where $a = \omega^2$ is a positive constant

Eq (5) can be written as two equations

$$\frac{d^4W(x)}{dx^4} - \beta^4 W(x) = 0 \quad (6)$$

$$\frac{d^2T(t)}{dt^2} + \omega^2 T(t) = 0 \quad (7)$$

Where

$$\beta^4 = \frac{\omega^2}{c^2} = \frac{\rho A\omega^2}{EI} \quad (i)$$

The solution of eq(7) can be expressed as

$$T(t) = A\cos\omega t + B\sin\omega t \quad (8)$$

The value of constant A and B in above equation can be calculated using initial boundary conditions. The solution of eq(6)

$$W(x) = Ce^{sx} \quad (9)$$

Where C and S are constants and derive the auxiliary eq as

$$s^4 - \beta^4 = 0$$

The roots of above eq are

$$s_{1,2} = \pm\beta,\ s_{3,4} = \pm i\beta \quad (10)$$

As a result, the equation's (6) solution is

$$W(x) = C_1e^{\beta x} + C_2e^{-\beta x} + C_3e^{i\beta x} + C_4e^{-i\beta x} \quad (11)$$

Where C_1, C_2, C_3, and C_4 are constant's, hence eq (11) can be written as

$$W(x) = C_1\cos\beta x + C_2\sin\beta x + C_3\cosh\beta x + C_4\sinh\beta x$$

$$W(x) = C_1(\cos\beta x + \cosh\beta x) + C_2(\cos\beta x - \cosh\beta x) + C_3(\sin\beta x + \sinh\beta x) + C_4(\sin\beta x - \sinh\beta x) \quad (12)$$

The boundary conditions of the simply supported beam can be used to find the constants C_1, C_2, C_3, and C_4 .

Equation(i) is used to calculate the fundamental frequencies of the beam.

$$\omega = \beta^2\sqrt{\frac{EI}{\rho A}} = (\beta l)^2\sqrt{\frac{EI}{\rho A l^4}} \quad (13)$$

3.1 Boundary Conditions For Pinned-Pinned Beam

$$\text{Deflection} = w = 0,$$

$$\text{Bending moment} = EI\frac{\partial^2 w}{\partial x^2} = 0$$

Thus eq(12) becomes

$$\sin\beta l = 0 \quad (14)$$

The value of $(\beta l)^2$ in eq(13) can be found by using eq(14) for pinned-pinned boundary conditions.

By using analytical method we will calculate first 3 natural frequencies of pinned-pinned beam length L=780mm and cross section area A= $(30\times5)mm^2$ made of wood having young's modulus E= 9.9× $10^9 N/m^2$, density ρ = 468kg/$m^3$and poission's ratio μ = .33 as shown in below table.

Table-2: Analytical results of beam using Euler-Bernoulli beam theory.

Mode	Analytical frequency(hz)
1	17.12
2	68.49
3	154.1

Table-3: Comparison of results between numerical and analytical approaches

Mode	Numerical frequency	Analytical frequency	(%) error
1	17.14	17.12	0.0002
2	68.561	68.49	0.00017
3	154.27	154.1	0.0017

5. EXPERIMENTAL ANALSYS OF FREE VIBRATION OF A PINNED-PINNED BEAM

A wooden beam having dimensions length L=780 mm and area A=30mm×5mm.The Pinned-pinned boundary conditions was used for experiment. In this experiment we will use universal vibration apparatus TM155,the end of the beam pinned-pinned support mounted on the testing machine and exciter frequency(0.8kg) which is electronically controlled placed at center of the beam.

Fig-6: Experimental setup of pinned-pinned beam

The experimental results for simply supported beam obtained are 16.45 HZ for first mode of vibration. Second and third resonating frequency are not in the range of exciter frequency(0-50HZ).

5. CONCLUSION

In this paper the numerical, analytical and experimental analysis of pinned-pinned beam are performed. The analytical and numerical results are found to have extremely good correlation in lower modes of vibrations. But as we go towards higher modes analytical and numerical results don't remain in good correlation. In this study its observed that the experimental result is having some reasonable error with analytical and numerical results because there is a discontinuity(hole) at middle of the beam where frequency exciter is attached and weight of frequency exciter which is 800gm may causes some error during experimentation.

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