Domination in the Corona of Jump Graph

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ABSTRACT: In this paper we characterized the dominating sets, Total dominating sets, and secure total dominating sets in the corona of two graphs. As direct consequences, the domination , total domination, and secure total domination numbers of these graphs were obtained.

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Keywords; graphs, domination ,total domination, corona.

1. Introduction:

Several results about domination number γ(J(G)) and total domination number γ_t(J(G)) are pper bounds in terms of the order n, minimum degree δ , maximum degree Δ, diameter diam(J(G)) of the graph J(G).Some of these results on domination number were obtained by Berge in [3], Dela Viña in[7], Flach and Volkmann I [8], Löwenstein and Rautenbach in [13], McCuaig and Shepherd in[14] and Volkman in[15]. Also some of the results on total domination number were obtained by Atapur and Soltankjhah in [1], Brigham et.al.in[4] Cockayne, et.al.in [5], Lam and Wei in[12], Haynes, Henning and Yeoin [9] [10] 1nd [11].

There are also types of domination in graphs which are being studied such as secure domination and independent domination. One of the most recent types of dominating set in a graph is the so-called *secure total dominating set.* This concept was introduced by Benecke et.al. [2] in 2007.

In this paper, we characterized the dominating, total dominating, and secure total dominating sets in the corona of two connected graphs. As qui9ck consequences, we determined the domination, total domination and secure total domination number of these graphs. To facilitate the results obtained, we need the following definitions.

Let J(G)= (V,E) be a connected simple graph and v∈ V(J(G)).The neighborhood of v is the set $N_{J(G)}$(v)=N(v)= { u ∈ V(J(G) :uv ∈ E(J(G)} If X⊆ V(J(G), then the open neighborhood of X is $N_{J(G)}$[X] = N[X] = X U N(X).

A subset X of V(J(G)) is a dominating set of J(G) if for every v∈ (V(J(G)) \ X) there exists x ∈ X such that xv ∈ E(J(G), i.e., N[X]=V(J(G). It is a total dominating set if for every u ∈ V(J(G))\X tfhere exists v ∈ X such that uv ∈ E(J(G) and [X\{v}]U{u} is a total dominating set, The domination number γ (J(G)) (total domination number γ_t (J(G)), or secure total domination number γ_{st} (J(G))) of J(G) is the cardinality of a minimum dominating (resp., total dominating secure total dominating)set of J(G).

The corona J(G) o J(H) of two jump graphs J(G) and J(H) is the graph obtained by taking one copy of J(G) of order n and n copies of J(H), and joining the i^{th} vertex of J(G) to every vertex in the i^{th} copy o J(H). For every v ∈ V(J(G)), denote by J(H^v) the copy of J(H) whose vertices are attached one by one to the vertex v. Subsequently, denote v+ H^v the sub graph of the corona J(G) o J(H) . Corresponding to the join ({v})+ H^v, v ∈ V(J(G)).

2. Domination in the corona of Jump graphs.

Theorem 2.1. Let J(G) be a connected graph of order m and let J(H) be any graph of ordrn . Then C ⊆ V(J(G) oJ(H)) is a dominating set in J(G0 0 J(H) if and only if V(v + H^v) ∩ c is a dominating set of v+H^v for any v ∈ V(J(G)).

Proof: Let C be a dominating set in J(G) 0 J(H) and let v ∈ V(J(G)). If v ∈ C hen {v} is a dominating set of v + H^v follows that V(v + H^v) ∩ C is a dominating set of v + H^v. Suppose that v ∉ C and let x ∈ C and let x ∈ V(v + H^v) \C with x ≠ v. Since C is a dominating set of J(G) o J(H), there exists y ∈ C such that xy ∈ E(J(G) o J(H)). Then y ∈ V(J(H^v)) ∩ C and xy ∈ E(v + H^v) This proves that V(v + H^v) ∩ C is a dominating set of v + H^v.

For the converse, suppose that V(v + H^v) ∩ C is a dominating set of v + $H^{v.}$ for every v ∈ V(J(G)). Then clearly C is a dominating set of J(G) o J(H)

Corollary 2.2 : Let J(G) is a connected jump graph of order m and let J(H) be any jump graph of order n. Then γ (J(G) o J(H)) = m
Poof: **Let C = V(J(G)). Then** V(v + H^v) ∩ C = {v} is a dominating set of v + H^v. for every v ∈ V(J(G)). By theorem 2.1, C is a dominating set of J(G) o J(H) hence , γ (J(G) o J(H)) ≤ | C | = m.

Next, let C^* be a minimum dominating set of J(G) o J(H). Then by Theorem 2.1

. V(v + H^v) ∩C^* is a dominating set of v + H^v for every v ∈ V(J(G)). It6 follows that

γ (J(G) o J(H)) = | C^* | ≥ m Therefore , γ (J(G) o J(H)) = m.

3. Total Domination in the corona of jump graphs.

Theorem 3.1: Let J(G) is a connected graph of order m and let J(H) be any graph of order n, Then C ⊆ V(J(G) o J(H)) is a total dominating set in J(G) o J(H) if and only if every
v ∈ V(J(G)),. Either

(i) V(v + H^v) ∩ C is a al dominating set of v + H^v or
(ii) v∈ C nd $N_{J(G)}$(v) ∩ C ≠ ϕ

Proof: Let C be a total dominating set in J(G) o J(H) and

v ∈ V(JG)). If V (v + H^v) ∩ C is total dominating set of v + $H^{v,}$ then we are done. So, suppose that V (v + H^v)∩ C is not a total dominating set of v + H^v. Suppose further that v ∉ C. Since C is a dominating set of J(G) o J(H), V(H^v) ∩ C mus be a dominating set of v + H^v Now since V (v + H^v) ∩ C = V (H^v) ∩ C is not a total dominating set of v + H^v . there exists u ∈ V (H^v) \C such that $N_{\mathbf{J(G)\ o\ J(H)}}$ (u) ∩ C = ϕ . This contradicts the fact that C is a total dominating set of J(G) o J(H). Thus, v∈ C. By assumption, V (v + H^v) ∩ C = {v} (otherwise,the set is total dominating set. Since C is total dominating set of J(G) o J(H) it follows that $N_{J(G)}$(v) ∩ C ≠ ϕ..

For the converse, suppose that the condition holds for C. Let x∈ V (J(G) oJ(H)) and let v ∈ V(J(G)) such that x ∈ V(v + H^v). consider the following cases,

Case 1: x = v

If x∈ C, then there exists u ∈ V (J(G)) ∩ (C \ {x}) Such that XU ∈ E (J(G) o J(H)) by (ii). If x ≠C, then V(H^v) ∩ c is a total dominting set of v + H^v by (i). Hence, there exists

y ∈ V (H^v)∩ C such that xy ∈ E (J(G) o J(H)).

Case 2. X ≠ v.

If v ∈ C, then xv ∈ E (J(G) o J(H)). If ∉ C, then there exists w∈ V (H) ∩ C such that xw ∈ E(J(G) o J(H)) byn (i)

In both the cases we have $N_{J(G)\ o\ J(H)}$ (x) ∩ C ≠ ϕ. Therefore, C is a total dominating of J(G) o J(H).

Corollary:3.2: Let J(G) be a conne ted graph of order m and let J(H) be any graph of order n. Then

γ_t(J(G) o J(H)) = m

Proof: Let c = V(J(G)). Then C is a total dominating set of J(G) o J(H) by Theorem 3.1 Thus

γ_t(J(G) o J(H)) ≤ |C| = m

Next let C^* be a minimum total dominating set of J(G) o J(H). Then, by Theorem 3.1

| V (v+H^v) ∩ C^* | ≥ 1 for every v ∈ V(J(G)). It follows that γ_t(J(G) o J(H)) = |C^*| ≥ m. There fore

γ_t(J(G) o J(H)) = m.

4. Secure Total Domination in the Corona of Jump Graphs.

Lemma 4.1: Let J(G) be a connected graph and let S be a secure total dominating set of J(G).Then S\{v} is a dominating set of J(G) for every v ∈ S. In particular $1 = \gamma$ (J(G)) ≤ γ_{st} (J(G)).

Proof: Let v ∈ S and let $S^* = S\backslash\{v\}$. Suppose S^* is not a dominating set of J(G). Then there exists

z ∈ V(J(G))\S^* such that zw ∉ E(J(G)) for all w ∈ S^* Then z ≠ v and v is the only element of S with

zv ∈ E(J(G)). However, the set (C\{v}) U {z} cannot be a total dominating set because zw ∉ E(J(G)) for all w ∈ S^*. This contradicts the fact that S is a secure total dominating set of J(G). Therefore, S\{v} is a dominating set of J(G). Moreover, if S is a minimum secure total dominating set of J(G), then the result implies that γ(J(G)) ≤ γ(J(G)) - 1.

Theorem 4.2: Let J(G) be a connected graph of order m and let J(H) be any graph of order n. Then

C⊆ V(J(G) o J(H)) is a secure total dominating set of J(G) o J(H) if and only if for every v ∈ V(J(G)), either

(i) V(J(H^v) ∩ C is a secure total dominating set of J(H^v) or
(ii) v∈ C and V(J(H^v)) ∩ C is a dominating set of J(H^v))

Proof: let C be a secure total dominating set J(G) o J(H) and let v ∈ V(J(G)). If V(J(H^v)) ∩ C is a secure total dominating set of J(H^v), then we are done. So suppose that V(J(H^v)) ∩ C ix noty a secure total dominating set of J(H^v) Suppose further that v∉ C. Since C is total dominating set of J(G) o J(H), V(J(H^v))∩ C must be a total dominating set of J(H^v). By assumption, there exists x ∈ V(J(H^v))\C such that [V(J(H^v))∩ C \{y}] U {x} is no a total dominating set for every y ∈V(J(H^v)) ∩ C with xy ∈ E(J(H^v). This implies that (C\{y}) U {x} is not a total dominating set of J(G) o J(H) for every y ∈ C with xy ∈ E(J(G) o J(H)). Contrary to our assumption of the set C. Therefore v∈C. If V(J(H^v))∩ C =ϕ and w ∈ V(J(H^v)), then (C\{v} u {w}) is not a total dominating set of J(G) o J(H),contrary to our assumption. This V(J(H^v)) ∩ C is a dominating set of J(H^v).

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