International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 08 Issue: 07 | July 2021 www.irjet.net p-ISSN: 2395-0072

FIRST ORDER SECOND MOMENT METHOD FOR STABILITY ANALYSIS OF A GRAVITY DAM

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Abstract - *Reliability study of dams is essential for operational safety, which depends on the complexity in dam failure causes. The assessment of the dam reliability can be mainly probabilistic or nonprobabilistic. The probabilistic method is usually applied to the cases with sufficient knowledge on dam parameters, while the nonprobabilistic method is suitable for the cases with insufficient knowledge on dam parameters. Since a dam can contain multiple parameters, information abundancy can vary among those parameters, and neither the probabilistic method nor the nonprobabilistic method alone is enough for dam reliability assessment. In this paper, the First order second moment method is used to analyse the dam reliability assessment. The probabilistic analysis is done along with the fuzzy method to determine the various factors that contributes to the safe operation of the dam.*

***Key Words*: Gravity Dam, Probabilistic Analysis, FOSM, Reliability index, Factor of Safety**

1. INTRODUCTION

A dam contains various parameters, so information abundancy can be unbalanced. Neither a single probabilistic model nor a single nonprobabilistic model is suitable for the dam failure risk assessment. As a result, a combination of probabilistic model and nonprobabilistic model should be studied to solve the risk analysis of dam failure under different operation conditions.

1.1 Probabilistic Risk Assessment

Malkawi et al. [1] established performance functions based on the Swedish circle method, simplified Bishop's method, simplified Janbu's method, and Spencer's method, respectively. The first-order second-moment method and the Monte Carlo simulation method were used to solve the reliability index, and the instability reliability model was established based on the reliability index. Kruger et al. [2] calculated the risk probability of the RCC gravity dam by using the first-order second-moment method, improved first-order second-moment method, and Monte Carlo simulation method and provided suggestions for maintenance and reinforcement of the dam based on the calculation results. Leszek [3] believed that there were various loads in the dam structure that maintained or caused the crash, and the uncertainty of the load ratio could be estimated. Based on this load ratio, the probability of dam instability was calculated. Leszek [4] assumed that the antisliding and sliding forces were controlled within the accracy range of 15%.

1.2 NonProbabilistic Risk Assessment

In the case of insufficient engineering data, Elishakoff [5] used the convex model for uncertainty analysis and verified the effectiveness of the method. Ben-Haim [6] considered the uncertainty of the variables and verified that the variation of the variables within a certain range had no great influences on the structure reliability. Elishakoff [7] raised the idea that the nonprobabilistic reliability index should be a specific interval rather than a value. Based on the properties of the convex set operation and considering the maximum allowable uncertainty of the system, Ben-Haim [8] proposed a theory of the nonprobabilistic reliability.

1.2 Study Area

The Hypothetical dam to be analyzed in this thesis is taken from the theme C of the eleventh ICOLD benchmark workshop on numerical analysis of dams (3IWRDD ICOLD, 2011). The problem in the benchmark workshop aims at analyzing the dam with a 2D model. Total height of the dam is 80 m with drain axis 10 m from the upstream end. The other dimensions are given in Figure 1. The objective of the ICOLD proposal was to benchmark numerical and analytical methods for the evaluation of the maximum sustainable reservoir level before dam collapsing and the evaluation of the uplift pressure distributions acting along the dambase.

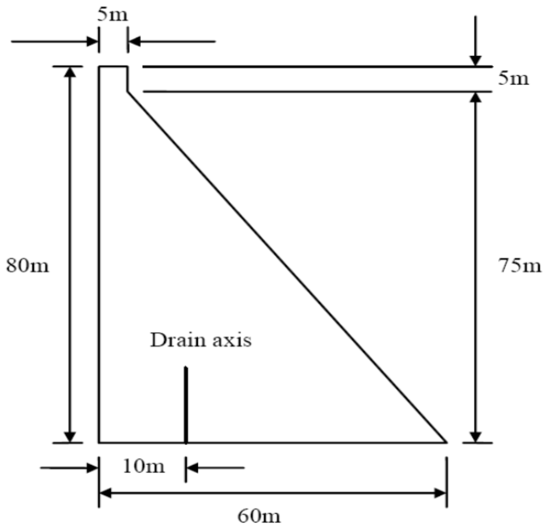

Fig -1: **Dam geometry (3IWRD ICOLD, 2011)**

1.3 Random Variables

The variables are classified as deterministic or random. In the present thesis, the random variables considered are friction angle 'φ' and cohesion 'c' along the dam-foundation interface. Probability density functions are determined for friction and cohesion and the probability distribution is assumed normal. The friction angle is defined by a normal probability function with a mean value μ_f and standard deviation σ_f. Similarly, cohesion is normally distributed with a mean μ_c and standard deviation σ_c.

Table -1: **Mean and standard deviation values used for friction and cohesion**

Normal Probability Distribution	Friction Angle (φ) Degrees	Cohesion (c) Mpa
Mean	52.4^0	0.3367
Standard Deviation	7.989	0.2468

1.4 Deterministic variables

The considered deterministic variables are
1. Concrete density (kN/m^3)
2. Water density (kN/m^3)
3. Water Pressure 'P' (kN/m^2)
4. Self-weight of the dam 'W' (kN/m)
5. Horizontal load due to water pressure acting on the upstream face of the dam 'H' (kN/m)
6. Uplift load acting on the base of the dam 'U' (kN/m)
7. Tensile stress 'σ' (kN/m^2)
8. Moment of inertia 'I' (m^4)

All the load variables are considered as deterministic. Some of the variables are considered constant throughout the calculations while some have their values determined according to the varying load conditions.

Two cases of drain effectiveness are considered with discrete probabilities associated:
Case A: Drains effective
Case B: Drains not effective

1.5 Stability Analysis

The Stability Analysis is carried out by deterministic method for the following load cases:
Load combination A: (Empty reservoir condition) : When the reservoir is empty, the force acting on the dam profile will be due to the self weight only, which acts at inner middle third. Other forces such as water pressure and uplift will be zero.
2. Load combination B (normal operating conditions): Full reservoir elevation, normal dry weather tail water, normal uplift, ice and silt (if applicable). Here for Indian conditions the silt load is neglected

3. Load combination C: (Flood discharge condition) - Reservoir at maximum flood pool elevation all gates open, tailwater at flood elevation, normal uplift, and silt (if applicable)

4. Load combination D: Combination of A and earthquake

5. Load combination E: Combination B, with earthquake but no ice

6. Load combination F: Combination C, but with extreme uplift, assuming the drainage holes to be Inoperative

7. Load combination G: Combination E but with extreme uplift (drains inoperative)

2.0 Methodology

2.1 Mean-Value First-Order Second-Moment Method (MFOSM)

The First-Order Second-Moment method is also referred to as the MFOSM method. The MFOSM method derives its name from the fact that it is based on a first-order Taylor's series approximation of the performance function linearized at the mean values of the random variables, and because it uses only second-moment statistics (means and standard deviations) of the random variables.(Wolff et. al., 2004)[9]

Let the performance function be written as :

$Z=g(X)=g(X_1, X_2, X_3, \ldots\ldots, X_n)$ 2.1

A Taylor's series expansion of the performance function about the mean value gives: $Z=g(\mu_x)$ where the derivatives are evaluated at the mean values of the random variables ($X_1, X_2, X_3, \ldots\ldots, X_n$) and μ_{xi} is the mean value of X_i. Truncating the series at the linear terms, we obtain the first-order approximate mean and variance of Z as:

$$\mu_Z = g(\mu_{x1}, \mu_{x2}, \ldots\ldots\ldots\ldots \mu_{xn}) \quad \ldots\ldots\ldots\ldots 2.2$$

and

$$\sigma_Z^2 = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^2 Var(X_i) + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j} CV(X_i X_j) \quad \ldots\ldots\ldots\ldots 2.3$$

where $CV(X_i.X_j)$ is the covariance of X_i and X_j

If the variables are uncorrelated, then the variance is:

$$\sigma_Z^2 = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^2 Var(X_i) \quad \ldots\ldots\ldots\ldots 2.4$$

The standard deviation of Z is

$$\sigma_Z = \sqrt{Var(Z)} \quad \ldots\ldots\ldots\ldots 2.5$$

Two approaches are used to estimate the variance of the performance function as approximated by the MFOSM method. The first method is a direct evaluation of the differential equation given in equation 2.3 or 2.4. This is a closed form solution for variance of the performance function. However, for most of slope stability analysis such evaluation is practically impossible and inconvenient. The second approach involves a numerical approximation of the partial derivatives. In this study, the second approach is used for computing the uncertainty of the performance function. Taylor series finite difference (TSFD) method is used for the numerical approximation of the partial derivatives and is as discussed below:

The performance function (Z) has to be evaluated at two points. The function is evaluated at one increment above and below the expected value of the random variable X_i as shown below:

$$Z_i^+ = g[\mu_{xi}, \mu_{x2}, \ldots\ldots\ldots (\mu_{xi}+\sigma_{xi}) \ldots\ldots] \quad \ldots\ldots 2.6$$

$$Z_i^- = g[\mu_{xi}, \mu_{x2}, \ldots\ldots\ldots (\mu_{xi}-\sigma_{xi}) \ldots\ldots] \quad \ldots 2.7$$

Although the derivative at a point is most precisely evaluated using a very small increment, evaluating the derivative over a range of plus and minus one standard deviation may better capture some of the nonlinear behavior of the function over a range of likely values. Using the central difference approximation

$$\frac{\partial g}{\partial X_i} = \frac{Z_i^+ - Z_i^-}{2\sigma_{xi}} \quad \ldots\ldots\ldots\ldots 2.8$$

Substituting the above equation in equation 2.4, we get

$$\sigma_Z^2 = \sum_{i=1}^{n} \left(\frac{Z_i^+ - Z_i^-}{2\sigma_{xi}}\right)^2 Var(X_i) \quad \ldots\ldots\ldots\ldots 2.9$$

If ($Z_i^+ - Z_i^-$) = ΔZ_i then,

$$\sigma_Z^2 = \sum_{i=1}^{n} \left(\frac{\Delta Z_i}{2}\right)^2 \quad \ldots\ldots\ldots\ldots 2.10$$

For correlated random variables

$$\sigma_Z^2 = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^2 Var(X_i) + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j} CV(X_i X_j) \quad \ldots\ldots\ldots\ldots 2.11$$

For dependent variables, $CV(X_i,X_j) = r_{ij}. \sigma_i . \sigma_j$

$$\sigma_Z^2 = \sum_{i=1}^{n} \left(\frac{\Delta Z_i}{2}\right)^2 + \frac{1}{2}\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \Delta Z_i \Delta Z_j r_{ij} \quad \ldots\ldots 2.12$$

2.2 Fuzzy Reliability Analysis

Reducing complex real-world systems into precise mathematical model always is the main trend in science and engineering. However, real-world situations are often not as deterministic as one has recognized. To deal with uncertainty, the probability theory has been used traditionally for statistical problems. This probability theory does not allow the subjectivity connected uncertainty to propagate in the analysis, as this estimates the mean and standard deviation of the measured samples. Moreover, the number of samples required is very large for any reasonable estimate of reliability of the system and it is seldom possible in civil engineering due to financial constraints. Furthermore, the uncertainties connected with geotechnical engineering may be non-random (subjective) in nature. Fuzzy set theory developed by Zadeh (1965) [10] has been applied widely to incorporate the non-random uncertainties into the developed fuzzy models

2.3 Fuzzy First-Order Second-Moment Method (FFOSM)

The uncertain parameters are described by their central values and standard deviations. In the present analysis, triangular fuzzy numbers (TFN) are considered. The value of the mode, minimum and maximum of the TFN are:

b = mode = E[X] = expected value,

a = E[X] - kσ[X] = minimum value, and

c = E[X] + kσ[X] = maximum value

k = the number of sigma units which will take values depending on data available and accuracy of the results desired.

In the present work k = 1.5 is considered. For multivariate problems involving M uncertain parameters the proposed method selects its points for function evaluation based on the α-cut or α-level concept of fuzzy numbers. The points for function evaluation in the parameter space are as explained in equation 2.14.

Vertex method algorithm has been applied in order to allow uncertainty to propagate through the solution processes. The vertex method, based on the α-level concept of a fuzzy number, essentially involves an interval analysis. The present study considers nine α-levels (from 0.1 to 0.9 possibility level) to represent the involved fuzzy numbers. For each investigated α level we need to evaluate the performance function 2n times, where n is the number of fuzzy uncertain variables.FS is the function of the uncertain variables as well as other non-fuzzy parameter, which will take single value while evaluating the FS. The following steps are used for the fuzzy reliability analysis of slope.

[1] The triangular fuzzy numbers for each of the input uncertain variables are constructed based on the amount of variability to be represented by the fuzzy sets. Thus the values of the parameters a, b and c for each of the fuzzy sets are obtained.

[2] The number of α-level are to be selected. For the case of three fuzzy variables, say X1, X2, X3, we will get six values of the uncertain parameters i.e., three lower bound values and three upper bound values at each α-level. According to the vertex method, we will get 2x3=6 points for the function evaluation. These six function evaluation points are obtained by the combination of the lower/ upper bound values and mean values of the uncertain parameters. The corresponding six points at each α-level are as shown below:

$$FS^{+}_{X1} = g[(X1^{+}_{\alpha i}), E(X2), E(X3)] \quad \text{...... 2.13}$$

$$FS^{-}_{X1} = g[(X1^{-}_{\alpha i}), E(X2), E(X3)] \quad \text{...... 2.14}$$

$$FS^{+}_{X2} = g[E(X1), (X2^{+}_{\alpha i}), E(X3)] \quad \text{......2.15}$$

$$FS^{-}_{X2} = g[E(X1), (X2^{-}_{\alpha i}), E(X3)] \quad \text{......2.16}$$

$$FS^{+}_{X3} = g[E(X1), E(X2), (X3^{+}_{\alpha i})] \quad \text{.....2.17}$$

$$FS^{-}_{X3} = g[E(X1), E(X2), (X3^{-}_{\alpha i})] \quad \text{......2.18}$$

Based on these six values of FS, the lower and upper bounds of FS can be chosen which represent the computed FS values as a fuzzy interval value i.e., FS_{max} and FS_{min} at that α-level. These interval values of FS constitute the resulting FS fuzzy set.

The expected value of FS is obtained by considering the modal values of the triangular fuzzy numbers of the uncertain variables.

The standard deviation σ(FS) is approximated by the Taylor series expansion of the function FS retaining only the linear terms. The general expression for the σ(FS)is given by:where,

n = number of uncertain soil parameter

$\sigma(X_i)$= Standard deviation of X_i

$CV(X_i, X_j)$ = Covariance between X_i and X_j.

The procedure adopted to calculate the standard deviation at each α-level is as given below:

Each parameter is sequentially set at bVi with the remaining parameters at their respective expected values. Each parameter combination gives one critical surface with FS_i . Hence, a total of 2^n values of FS are obtained at each α- level.

Now, standard deviation for uncorrelated variables is given by

$$\sigma(FS_{\alpha i}) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial FS_{\alpha i}}{\partial X_i}\right)^2 \sigma^2(X_i)} \quad \text{............ 2.19}$$

Using Central difference approximation

$$\frac{\partial FS_{\alpha i}}{\partial X_i} = \frac{FS^{+}_{\alpha i} - FS^{-}_{\alpha i}}{2V_{at\,\alpha=0.5}} \quad \text{............ 2.20}$$

substituting the above equation in equation, we get

$$\sigma^2(FS_{\alpha i}) = \sum_{i=1}^{n}\left(\frac{FS^{+}_{\alpha i} - FS^{-}_{\alpha i}}{2k_{at\ \alpha=0.5}}\right)^2 \quad \text{............ 2.21}$$

The overall estimated values of standard deviation of FS is evaluated using

$$E(\sigma(FS)) = \frac{1}{N}\left[\sum_{i=1}^{N} .\sigma(FS_{\alpha i})\right] \quad \text{............ 2.22}$$

Knowing the expected value and the standard deviation of the performance function one can calculate the probability of failure after assuming a suitable distribution for the performance function and the reliability index. Furthermore, with the FS_{max} and FS_{min} values at each α -level, the FS fuzzy set can be constructed.

3.0 Results and Discussions

Table 3.1 Results of FOSM Method for Various Load Combinations

Load Combination	β	Probability of failure pf
B	2.17	0.015
C	2.17	0.015
D	3.406	0.003
E	2.07	0.0192
F	1.97	0.0224
G	1.86	0.0314

Table 3.2: Results of FOSM Different Water levels – Drains Operative and No seismic Load

Water Level H in m	β	Probability of failure pf
75	2.381	0.0087
76	2.23	0.0129
77	2.261	0.0119
78	2.29	0.011
79	2.202	0.0139
80	2.23	0.0129

Table 3.3: Results of FOSM for shear failure for Different waterlevels with Seismic Loads – Drainage gallery Operative

Water Level in m	β	Probability of failure pf
75	2.064	0.0197
76	1.914	0.0281
77	1.939	0.0262
78	1.965	0.025
79	1.873	0.0307
80	1.897	0.0294

Table 3.4: Results of FOSM method for different water levels – Drainage gallery Inoperative

Water Level H in m	β	Probability of failure pf
75	1.974	0.0244
76	1.83	0.0336
77	1.855	0.0322
78	1.879	0.0307
79	1.778	0.0384
80	1.801	0.0359

Table 3.5: Results of FFOSM method for sliding failure for different IS load combinations

Load Combination	β	Probability of failure pf
B	1.782	0.0375
C	1.838	0.0336
D	3.23	0.0006
E	1.129	0.1292
F	0.273	0.3947
G	0.81	0.2090

Table 3.6 : Results of FFOSM Method for Sliding failure Different Water levels without considering seismic load and Drainage effective.

Water Levels (m)	β	Probability of failure pf
70	1.159	0.123
72	1.008	0.1587
75	0.719	0.2389
78	0.53	0.3015
80	0.361	0.3594

Table 3.7 : Results of FFOSM method for Sliding failure Different Water levels without considering seismic load and Drainage Ineffective.

Water Levels (m)	β	Probability of failure pf
70	0.715	0.2450
72	0.749	0.2266
75	0.783	0.2177
78	0.502	0.3085
80	0.252	0.4013

Table 3.8 : Results of FFOSM for Sliding failure Different Water levels considering seismic load and Drainage effective.

Water Levels (m)	β	Probability of failure pf
70	1.159	0.1230
72	1.008	0.1587
75	0.774	0.2206
78	0.53	0.3015
80	0.575	0.2860

Table 3.9 : Results of FFOSM for Sliding failure Different Water levels considering seismic load and Drainage Ineffective.

Water Levels (m)	β	Probability of failure pf
70	0.782	0.2177
72	0.618	0.2723
75	0.363	0..3594
78	0.099	0.46
80	-0.083	0.5319

4. CONCLUSIONS

From the Above Tables, it can be seen that the Reliability index and the corresponding Probability of failure pf for different load combinations for the dam section taken. It is observed that as the reliability index increases, the probability of failure β decreases and vice versa.

A Comparision is made between the Probabilistic first order Second Moment method and fuzzy first order second moment method for different water levels and and IS Load combinations. The probabilistic FOSM method gives a higher value of Reliability index as compared to the FFOSM method. Further It is seen that the value of reliability index is lesser in case of Load Combination G in the hypothetical dam section as well as the dam section of the study area which is the combination of seismic load with the drainage being inoperative. It is also seen that the value of reliability index decreases as the water level in the dam increases. The highest value of the reliability index was obtained in case of Load combination D which is the case of empty reservoir with earthquake forces

REFERENCES

[3] A. I. H. Malkawi, W. F. Hassan, and F. A. Abdulla, "Uncertainty and reliability analysis applied to slope stability,"Structural Safety, vol. 22, no. 2, pp. 161–187, 2000.

[4] C. M. Kruger, A. C. Neto, and D. A. V. Kruger, "Structural reliability analysis and monitoring program applied to a roller-compacted concrete dam," in Proceedings of the 1st International Symposium on Life-Cycle Civil Engineering, Life-Cycle Civil Engineering, Varenna, Italy, June 2008.

[5] O. Leszek, "The risk of dam stability loss," in Proceedings of the International Congress on Large Dam, Beijing,China, October 2000.

[6] O.Leszek,"The risk of dam instability,"Express Water Resources & Hydropower Information, vol. 22, no. 5, pp. 32-33, 2001.

[7] I. Ellishakoff, "Essay on uncertainties in elastic and viscoelastic structures from A M Freudenthal's criticisms to modern convex modeling,"Computers&Structures, vol. 56, no. 6, pp. 871–895, 1995.

[8] B.-H. Yakov, "A non-probabilistic concept of reliability,"Structural Safety, vol. 14, no. 4, pp. 227–245, 1994.

[9] I. Elishakoff, "Discussion on: a non-probabilistic concept of reliability,"Structural Safety, vol. 17, no. 3, pp. 195–199, 1995.

[10] Y. Ben-Haim, "Robust reliability of structures,"Advances in Applied Mechanics, vol. 33, pp. 1–41, 1997.

[11] Wolff, T. F., Demsky, E. C., Schauer, J. and Perry, E. 1996. Reliability assessment of dike and levee embankments, in Uncertainty in the geologic environment: from theory to practice, Proceedings of Uncertainty '96, (C. D. Shackelford, P. P. Nelson, and M. J. S. Roth), 636–650, ASCE Geotechnical Special Publication No. 58

[12] L.A. Zadeh, "Fuzzy Sets,"Information and Control, June 1965, pp. 338-353.