

DESIGN, SIMULATION AND RESPONSE ANALYSIS OF FIR FILTER DESIGNED BY WINDOWING TECHNIQUE

Abbati Muhammad Ado¹, Jitendra Vaswani², Muhammad Ibrahim Zubair³

¹PG Student, Electronics and Communication Engineering Department, Mewar University, Chittorgarh, India

²Assistant Professor, Electronics and Communication Engineering Department, Mewar University, Chittorgarh, India

³PG Student, Electronics and Communication Engineering Department, Mewar University, Chittorgarh, India

ABSTRACT: Digital filters play a very important role in digital signal processing (DSP), digital filters are systems that operate on sampled, quantized input signals (digital signals) to allow certain range of frequency to pass either low frequency or high frequency, it can also pass or reject certain band of frequency from the given input signal bandwidth, or to achieve other filtering objectives. Finite impulse response (FIR) filter and infinite impulse response (IIR) are the two types of digital filters used, One of the methods that are used for the design of FIR filter is the windowing method, this method uses a window function such as rectangular, triangular, hamming, hanning, and Kaiser Window to truncate the infinite impulse response (e.g. Sinc signal) in order to get the finite impulse response. In this work a low pass FIR filter is designed using four different window functions namely Rectangular, Hamming, Hanning, and Blackman. Analysis and comparison on the frequency response of each low pass FIR filter designed is done, so that conclusion can be made on the window function that gives good filter response.

Keywords: filter, frequency, FIR, IIR, window functions.

1.0 INTRODUCTION

1.1 FILTER

A filter is an electronic circuit that changes the wave shape, amplitude-frequency or phase-frequency characteristics of a given signal, for the purpose of removing noise, extracting information from the signal, or to separate two or more signals that combine together. Filters played an important role in the field of engineering especially electrical, electronics and communication fields of engineering. Filters generally are frequency selective circuit that allow only certain range of frequency to pass either low frequency or high frequency, Filters are generally categorized into analog filters and digital filters. [1]

1.2 ANALOG FILTER

Analog filter is a type of filter that operates on continuous-time signal to remove noise from the signal, to allow certain range of frequency to pass or block it, or to achieve any filtering objectives. Analog filter circuits are made of analog components such as resistors, capacitors, inductors, and op amps. They are also characterized by their continuous impulse response described by a differential equation. Laplace transform is used to represent the transfer function of the analog filter [3]. Types of analog filters are.

- i. Low pass filter (LPF)
- ii. High pass filter (HPF)
- iii. Band pass filter (BPF)
- iv. Band reject filter (BRF)
- v. Notch filter
- vi. Resonance filter
- vii. All pass filters.

1.3 DIGITAL FILTER

A digital filter is a type of filter that operates on a digital input signal and produced a digital output signal for filtering purposes. It is an algorithm implemented in hardware or software that plays an important role in digital signal processing (DSP) systems below. [1]

- i. Speech signal processing
- ii. Image processing
- iii. Data transmission
- iv. Data compression
- v. Biomedical signal processing
- vi. Digital audio signal processing

Digital filters operate on sampled, quantized input signals (digital signals) to allow certain range of frequency to pass either low frequency or high frequency, it can also pass or reject certain band of frequency from the given input signal bandwidth, or to achieve other filtering objectives

[3]. Finite impulse response (FIR) filter and infinite impulse response (IIR) are the two types of digital filters used depending on the requirement of the system or application in which the filter is to be used.

1.3.1 Types of digital filters

i. Finite impulse response (FIR) filter

This is a type of digital filter in which its impulse response is of finite duration. [1] This filter can be represented by the difference equation below.

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k)$$

From the difference equation of the FIR filter above, the impulse response $h(k)$ is of finite values from 0 to N , the equation also indicates that the output of the filter $y(n)$ depends on the present and past values of the input signal. Now the transfer function of the FIR filter can be obtained by taking the z-transform of the impulse response as below.

$$H(z) = \sum_{k=0}^{N-1} h(k)z^{-k}$$

The diagram below is block diagram of digital FIR filter, with the input, output, delays, coefficients, and sum.

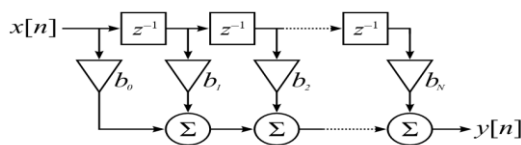


Fig. 1.0 FIR filter block diagram

ii. Infinite impulse response (IIR) filters

This is a type of digital filter in which its impulse response is of infinite duration, the input and output signals to IIR filter are related by the convolution sum below.

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

Now the problem with the equation above is that it cannot be implemented by computation because of the infinite impulse length that started from zero to infinity, instead the equation is expressed in a recursive form which is called the difference equation of the IIR filter as given below.

$$y(n) = \sum_{k=0}^N b_k x(n-k) - \sum_{k=1}^M a_k y(n-k)$$

Where a_k and b_k are the coefficients of the IIR filter.

From the difference equation above, the output signal $y(n)$ depends on the present and past values of the input signal as well as past output values, this shows that the IIR filter has a feedback system. [1]

2.0 LITERATURE REVIEW

Various articles and research papers were reviewed which formed the literature review of the thesis. However for the purpose of this publication some part of the literature review is discussed as below, Manjinder Kaur¹ and Sangeet Pal Kaur in their research paper "FIR low pass filter designing using different window functions and their comparison using MATLAB" the simulation result shows that the triangular window produced inefficient low pass filter result compared to other window functions, hence should not be chosen for efficient FIR low pass filter design. Among all the window functions the Blackman Harris window is the best choice for FIR filter design using window functions because it produces less side lobes and zero leakage factor. [4]

In another research work done by N. M. Shehu, A.S. Gidado, and M.I. Faruk on "response analysis of FIR high pass filter design using window methods" they make a comparative analysis based on the magnitude response, phase response, and pole-zero plots of the FIR high pass filter designed using three window functions namely: Rectangular, Kaiser, and Tukey window functions. The simulation result shows that the rectangular window function is the best for the design of FIR high pass filter because its magnitude response is close to the specified cut-off frequency. [5] [6]

Sumbal Zahoor and Shahzad Naseem in their research paper worked for an optimized and efficient design of digital FIR band pass filter from software to hardware implementation using Hamming, Hanning, Blackman, and Kaiser as the window functions. The Kaiser window function is the best choice based on the simulation result as it showed less transition band and minimum main lobe width, based on the same specifications ($N = 15$, $\beta = 0.5$, $fc1 = 0.4$, $fc2 = 0.5$) given for the design of the FIR band pass filter using the mentioned window functions above. And for realization of the transfer function the best-chosen structure is the direct-form structure, this design method reduces arithmetic complexity and hardware resources, hence produced optimized FIR band pass filter. [7, 8]

In the work of Er. Sandeep Kaur, and Er. Sangeet Pal Kaur, on "Design of FIR filter using hanning window, hamming window and modified hamming window". Based on the simulation result of their work, the FIR filter designed using the modified coefficients of the hamming window

function give a better and efficient design than the hanning and hamming window functions, because it provided smaller main lobe width, and sharp transition band. [9]

Also, in a published paper by Mr. Ankan Bhattacharya on FIR filter design, a modified window function and a hamming window function were used in the design, a frequency response of low pass, high pass, band pass, and band stop FIR filter of the modified window function and the hamming window function were shown in the simulation result, by comparison the modified window function give efficient and best design. [10]

3.0 METHODOLOGY

A MATLAB software was used for the design and simulation of the low pass FIR filter, the codes of the design are implemented on the software, and simulation result of various plots of the low pass FIR filter designed were observed. At the first stage of the design methodology, the design specifications of the filter such as order, range of values, cut off frequency, angle, etc. were given. A sink function was define and plotted with it frequency response, rectangular window function was defined and plotted as well, truncation of the sink function was done by multiplying it with rectangular window function, the frequency response of the multiplication result is the low pass FIR filter designed using rectangular window function by windowing technique. Subsequently a different window function Hamming was used, the result is the low pass FIR filter designed using Hamming window function, the other two window functions used are Hanning and Blackman window functions.

A noisy sine signal was defined with its time domain plot. The low pass FIR filter designed above using each window function was called, and used to filter the noisy sine signal above, the time domain of the noisy signal before and after filtering were plotted, the four different filtering result were compared, and conclusion was made on the filter that gives best filtering result.

3.1 DESIGN SPECIFICATIONS

The parameters or specifications used in the design of the low pass FIR filter are:

- i. Order of the filter $m = 50$
- ii. Range of values $n = 0:m-1$
- iii. Sampling frequency $f_s = 2000$
- iv. Cuff-off frequency $f_c = 0.1$
- v. Angles $\theta_1, \theta_2, \theta_3$
- vi. Frequency of a noisy sine signal $f_1 = 50$
- vii. Time domain t

3.2 DIGITAL FIR FILTER DESIGN METHODS

- i. Frequency sampling method
- ii. Chebyshev approximation method
- iii. Windowing Method

3.3 FIR FILTER DESIGN USING WINDOWING TECHNIQUE

The design of FIR filter using windowing method involved the use of window function such as rectangular, hamming, hanning, and Kaiser Window and multiplies it with the desired infinite impulse response to obtain the finite impulse response of the filter, this truncation produces ripples and overshoot both in the passband and the stopband of the FIR filter frequency response. This ringing effect near the band edge of the FIR filter is called Gibbs phenomenon, the magnitude of this Gibbs phenomenon differs for each window function used in the design. All the window functions have different properties in time and frequency domain, having different side lobe attenuation and transition width. The best window function to be choose for the design of FIR filter is one that provide good filter response with reduced side lobes and comparatively less pass-band and stop-band ripples.[11]

Generally, in the design of any digital filter, the desired frequency response of the digital filter response $H_d(e^{j\omega})$ is given below.

$$H_d(e^{j\omega}) = \sum_{-\infty}^{\infty} h_d(n)e^{-j\omega n}$$

By taking the inverse discrete Fourier transform (IDFT) of the desired frequency response an infinite impulse response $h_d(n)$ can be obtain below.

$$h_d(n) = 1/2\pi \int_{-\infty}^{\infty} H_d(e^{j\omega})e^{j\omega n} d\omega$$

Unfortunately, this impulse response $h_d(n)$ is infinite and hence lead to unrealizable IIR filter. However, this infinite impulse response can be made finite by truncating it, this truncation is achieved by multiplying the infinite impulse response with a window function $w(n)$ to get the finite impulse response $h(n)$. [12]

$$h(n) = h_d(n) \cdot w(n)$$

Now the design frequency response of the FIR filter is obtained from the finite impulse response $h(n)$ as.

$$H(e^{j\omega}) = \sum_{n=0}^{m-1} h(n)e^{-j\omega n}$$

Also, the transfer function of the FIR filter can be determined by taking the Z-transform of the impulse response as. [13]

$$H(z) = \sum_{n=0}^{m-1} h(n)z^{-n}$$

3.4 WINDOW FUNCTIONS

Window functions are used to truncate the infinite impulse response (e.g sinc function) in the design of FIR filter, some of the window functions are discuss below. [14]

i. Rectangular window function

$$W_R(n) = \begin{cases} 1, & 0 < n < L - 1 \\ 0, & \text{otherwise} \end{cases}$$

ii. Hanning window function

$$W_{HN}(n) = \begin{cases} 0, & \text{otherwise} \\ 0.5 - 0.5 \cos\left(\frac{2\pi n}{L-1}\right), & 0 < n < L - 1 \end{cases}$$

iii. Hamming window function

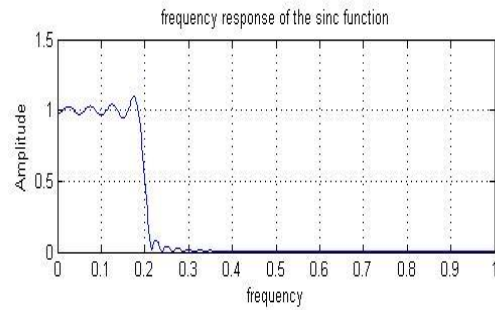
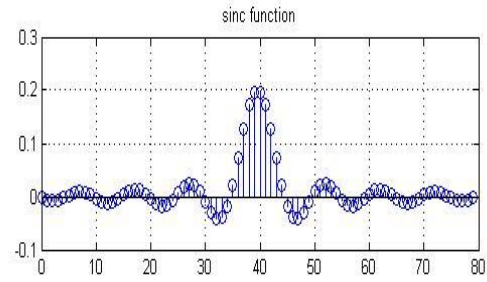
$$W_H(n) = \begin{cases} 0, & \text{otherwise} \\ 0.54 - 0.46 \cos\left(\frac{2\pi n}{L-1}\right), & 0 < n < L - 1 \end{cases}$$

iv. Blackman window function

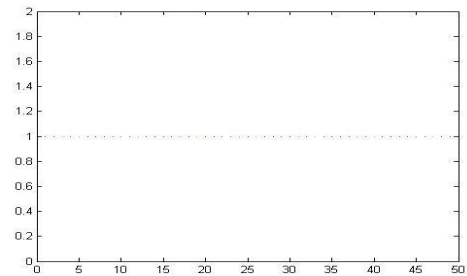
$$W_B(n) = \begin{cases} 0, & \text{otherwise} \\ 0.42 - 0.5 \cos\left(\frac{2\pi n}{L-1}\right) + 0.08 \cos\left(\frac{4\pi n}{L-1}\right), & 0 < n < L - 1 \end{cases}$$

4.0 SIMULATION RESULT AND ANALYSIS

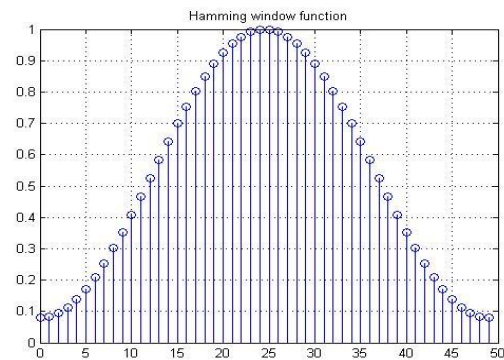
There are various plots from the simulation result using the MATLAB software, the plot of the sink signal, window functions, response of the low pass FIR filter designed, noisy sine signal, as well as the filtered sine signal are shown below.



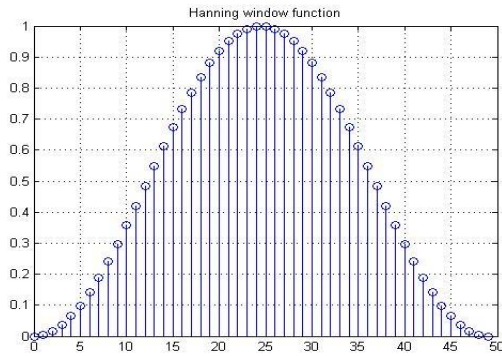
a) Time domain and frequency response of the sink function.



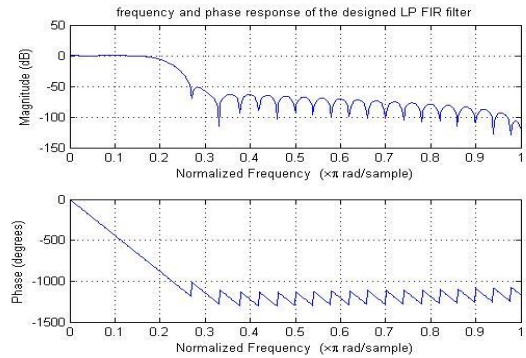
b) Plot of Rectangular window function



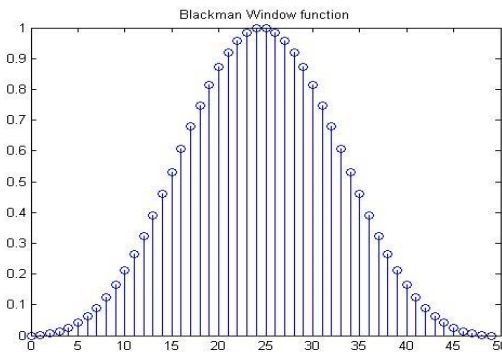
c) Plot of hamming window function.



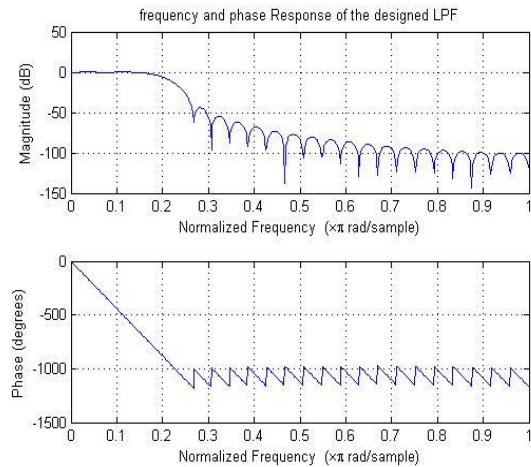
d) Plot of Hanning Window function



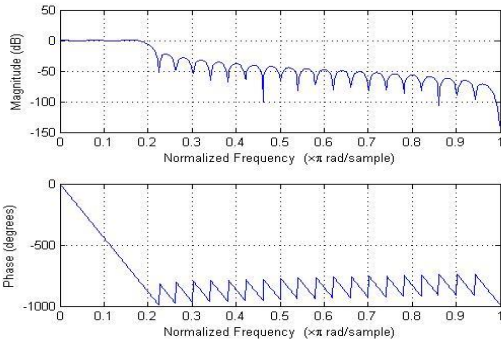
g) Response of low pass FIR filter designed using Hamming window function.



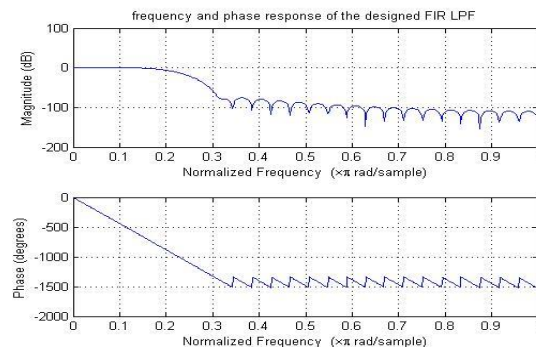
e) Plot of Blackman window function



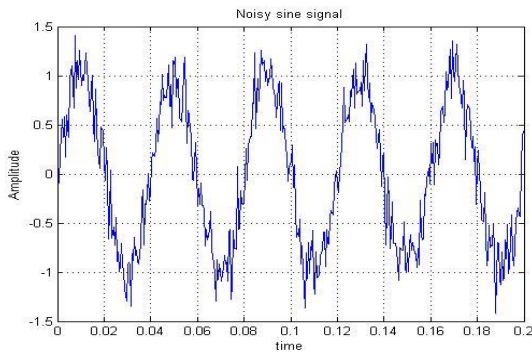
h) Response of low pass FIR filter designed using Hanning window function.



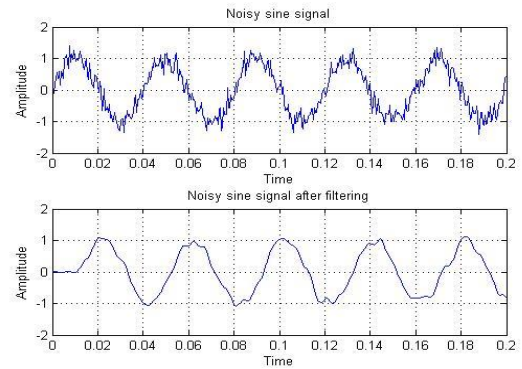
f) Response of low pass FIR filter designed using Rectangular window function.



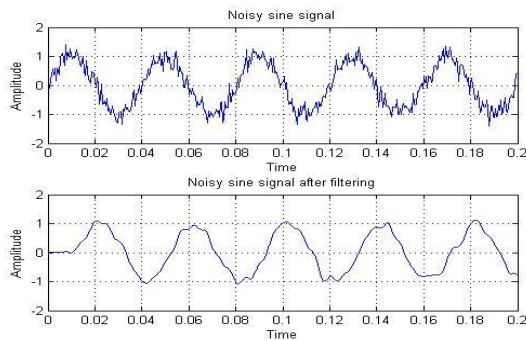
i) Response of low pass FIR filter designed using Blackman window function.



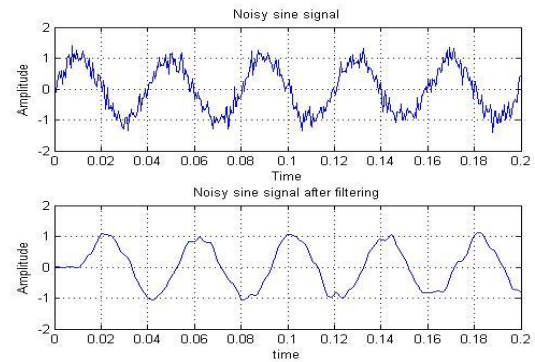
j) Noisy sine signal.



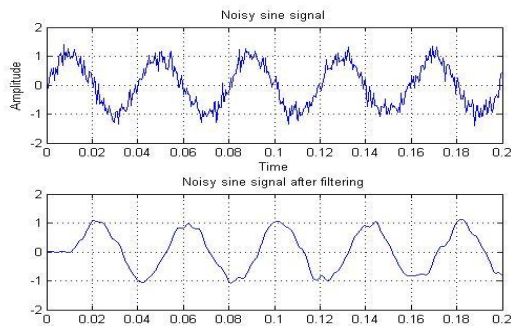
m) A comparative plot using Hanning windowed function filter



k) A comparative plot of Noisy sine signal with the filtered noisy signal using Rectangular Windowed function filter.



n) A comparative plot using Blackman windowed function filter.



l) A comparative plot using Hamming windowed function filter

4.1 ANALYSIS ON THE DESIGNED FIR FILTER RESPONSES

All the window functions Triangular, Rectangular, Hanning, Hamming and Blackman have completely different properties in time and frequency domain, having different side lobe attenuation and transition width. The best window function to be choose for the design of FIR filter is one that provide accurate type of responses with reduced side lobes and comparatively less pass-band and stop-band ripples. Considering the frequency response of the low pass FIR filter designed using rectangular window function in (f), there is a sudden transition at around 0.2 from passband to stopband region, this make the transition width very small, the stopband also consist of ripples with high side lobes as shown in the diagram. In the case of the frequency response of the low pass FIR filter designed using Hamming window in figure (g), transition from the passband to stopband start at around 0.2, with transition width 0.2 to 0.3, and a high side lobes

in the stopband region as shown above. Also, for the response of FIR filter designed using Hanning window function in figure (h), there is almost similar result with that of FIR low pass filter designed using Hamming window function, due to the slight difference in the formula of Hanning and Hamming window functions.

However, looking at the frequency response of the FIR low pass filter designed using Blackman window function in (i), transition from the passband to stopband start after 0.2 and stop after 0.3, there is also reduced in the width and height of the side lobes in the stopband, unlike that of the FIR filter designed using rectangular, Hanning, and Hamming window function that have high stopband ripples.

5.0 CONCLUSION

Based on the simulation results and its analysis, the design of low pass FIR filter using windowing technique on MATLAB software, four different window function namely Rectangular, Hamming, Hanning, and Blackman were used for the design using windowing technique, based on the analysis of the four filter responses designed, and also looking at the comparative plots of the noisy sine signal filtered using the four filters designed, the Rectangular

Although looking at the diagrams from (k) to (n), there are four comparative plots of the noisy sine signal and its filtered form using the four different low pass FIR filter designed using the four window functions, the four comparative plots almost give similar result, even though, we expect to see Blackman window function to give a better filtering result than the other three window functions. However, this depends on the nature and properties of the signal to be filtered, and which filter specifications meet the requirement or values of the signal to be filtered, such analysis need to be done before designing or chosen a filter so as to avoid filter mismatch or removing part of the signal that is not supposed to be remove.

window function is not suitable for the FIR filter design based on this specifications, because it produces filter response with less transition width and high side lobes ripples in the stopband. The Hamming and Hanning window functions almost gave similar result due to the similarity in their formula. However, Blackman window function gave better filter response than that of rectangular, Hamming, and Hanning window functions, as it produces filter response of smaller width and height of the ripples in the stopband

REFERENCES

- [1] Emmanuel C. Ifeakor, Bamie W. Jervis: "Digital Signal Processing, A Practical Approach", pp 318-398, PEARSON, second edition
- [2] Sanjit K. Mitra. "Digital Signal Processing, a computer-based approach 4th edition McGraw Hill," pp 489-528.
- [3] P. Dutilleux, U. Zolzer, "Chapter 2 Filters", pp 33, 2002 John Wiley & Sons, Ltd
- [4] Manjinder Kaur, Sangeet Pal Kaur, "FIR Low Pass Filter Designing Using Different Window Functions and their Comparison using MATLAB", International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, Vol. 5, Issue 2, February 2016.
- [5] N. M. Shehu, A.S. Gidado, M.I. Faruk, "Response analysis of FIR high pass filter design using window methods", International Journal of Engineering Applied Sciences and Technology, 2019 Vol. 4, Issue 5, ISSN No. 2455-2143, Pages 30-35.
- [6] Shehu, NM; Baraya, JT; Sani, MH, "Comparative Analysis of a High Pass Finite Impulse Response (FIR) Filter Designed and Simulated by Window Methods", J. Appl. Sci. Environ. Manage. Vol. 23 (4) 721-725 April 2019.
- [7] Sumbal Zahoor and Shahzad Naseem, "Design and implementation of an efficient FIR digital filter", Electrical & Electronic Engineering research article, Cogent Engineering (2017), 4: 1323373.
- [8] Mohammed Mynuddin, Md. Tanjimuddin, Md. Masud Rana, Abdullah, "Designing a Low- Pass Fir Digital filter by using Hamming window and Blackman window Technique", Science Journal of Circuits, Systems and Signal Processing, Vol. 4, No. 2, 2015, pp. 9-13

[9] Er. Sandeep Kaur, and Er. Sangeet Pal Kaur, "Design of FIR filter using hanning window, hamming window and modified hamming window", International Journal of Advanced Research in Computer Engineering & Technology (IJARCET) Volume 4 Issue 5, May 2015.

[10] Mr. Ankan Bhattacharya, "A modified window function for FIR filter design with an improved frequency response and its comparison with the Hamming window", International Journal of Science, Engineering and Technology Research (IJSETR) Volume 2, Issue 5, May 2013

[11] Sanjay Sharma, "Digital Signal Processing", Katson Books.

[12] S Salivahanan, A Vallavaraj, C Gnanapriya: "Digital Signal Processing", pp 381-401, Tata McGraw-Hill, second edition, 2010.

[13] John G. Proakis, Dimitris G. Manolakis, "Digital signal Processing" PEAESON, 4th edition.

[14] Su Peng, "Design and analysis of FIR filters based on Matlab", Degree Project of Bachelor thesis, Linnaeus University, school of computer science, physics and mathematics.