

Robust Stability for Time Delayed SISO Systems Using Contoured Robust Bode Plots

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Abstract: Plants, which have delayed output with respect to input reference signal, are called as time delay systems due to their specific phase response characteristics. They have constraints, such as bandwidth limitation, achievable sensitivity reduction etc. In the design of feedback control systems the conventional control methods cannot be directly applied to time delay systems due to internal stability problem. A new method of the robust control problem of time delay systems are stable parameters of PID controller from the tuning methods and contours controller design using contoured robust bode (CRC Bode) plots.

Key words – CRC Bode, RHP zero, SISO systems, NPM, Z-N tuning, RS, RP, NP

I. Introduction

Controller design for time delay systems plays an important role meanwhile many of the industrial plants or process (such as electrical systems, mechanical systems and electromechanical systems) exhibits delayed response at outputs in response to the inputs and also have region of in-stable performance index. The time delay systems are responsible for control complexity and degrade the system performance under disturbance conditions. To regulate the degree of performance index, robust controllers are needed. In this paper Contoured Robust Controller Bode (CRC Bode) plot is analyzed with the help of weight-sensitivity functions and used for frequency domain robust controller design for a single input single output time delay system. The CRC Bode plot is based on robust the performance measure by satisfying robust stability and nominal performance for all frequencies. Which represents the margins of the transfer function in graphical manner; the contours specify a degree of RP, NP and define acceptable and forbidden regions for the controller frequency response.

Random selection of weights are used to design a robust controller of different types of SISO system with respected sensitivity functions avoids the region of plot whose performance index greater than one. The UNCERTAINTY FOR ROBUST STABILITY is the difference between actual control system and model of the control systems referred as model mismatch or model uncertainty this model is used for design a controller for robust stability. Any model of the system contains uncertainty are represented in time domain as well as

frequency domain that is parametric uncertainty and dynamic uncertainty.

This paper robust stability of the system is measured by proceeding following approach Step1, mathematical model of model uncertainty that is determine the uncertain set. Step2 robust stability of system, which measures the whether the system remains stable for all the uncertain set. Step 3, if robust stability is satisfied then we need to measure the performance criteria for a closed loop system to exhibit desired performance for all control point of view this can be mathematically expressed as the product of uncertain weights and sensitivity function. Therefore sensitivity function defined by the all uncertainty within a certain limited uncertainty bound as to be enclosed by inverse of the performance weight. Than the system is said to be obeying the performance requirements and similarly the stability requirements are defined in terms of weight and sensitivity functions of a closed loop system. Another way of the same concept can expressed using the contour bode plots. Contour bode plots are able to assess the controller robustness by formulating a forbidden regions through which the designed controller should not passing if it is passing through forbidden region the controller needs to be redesign are the parameters should be vary.

In this paper robust stability of time delayed systems, Ziegler Nichols tuning methods are used for finding proportional integral derivative controller gain constants $K_p K_d K_i$, the loop shaping CRC Bode contours [reference 1] paper uses classical tuning methods for finding controller parameters this may take more time for continuous evolution of controller parameter with respect to desired frequency domain characteristics of controller.

The robust stability for time delayed systems are difficult to measure robustness using methods like uncertainty H-infinite modeling uncertainty test etc. Because these systems are need to remodel with changed uncertainty since these systems are sensitive to the perturbations therefore this paper is used for better understanding of analyzing robustness in sense of visualized contours of time delayed control systems.

II. Contour Robust controller Bode(RCR Bode) Plots

The CRC Bode plot measures whether or not a controller satisfies a robust performance criteria and also how sensitive the criteria is to changes in gain and phase. "COUNTERED ROBUST CONTROLLER BODE" plot are new methods to select the weight sensitive functions of a given system transfer function, Therefore the contours are not passing through the forbidden region for all frequencies, that implies that the RS and RP criteria is satisfied. Control systems are robust if they satisfactory stability and performance characteristics, Common classical design objectives of gain and phase margin are direct measure of system robustness But classical techniques are not well suited to the problem of meeting specific robust stability and performance without some trial and error therefore, Robust stability and robust performance is possible, if to place the bounds on the bode magnitude and phase plots Which gives manual loop-shaping controller design it indicate when certain robustness objectives have been reached, Standard loop-shaping bode design approach for SISO systems are used only in minimal phase systems (no pole/zero cancellation). Robust control of SISO system (NMP systems) is obtained by proper selection of weight sensitivity functions.

Let the transfer function of NMP system is,

$$P(s) = \frac{\text{RHP-ZERO}}{\text{PLOE}}$$

Where, P(s) is open-loop transfer function. L = PC, loop transfer function, here C is the transfer function of PID controller

$$C = Kp + Ki * \frac{1}{s} + Kp * s$$

Kp, Kd, Ki Are calculated in Matlab Simulation.

Consider the performance in terms of the weighted sensitivity function and complementary weight sensitive function Assume closed loop transfer function is nominally stable

$$S(s) = 1 / (1+L(s)), \text{ sensitivity function} \quad [1]$$

$$T(s) = L(s) / (1+L(s)), \text{ complimentary sensitivity} \quad [2]$$

The NP, RS, RP determine the robust stability of the system

$$|Wp(s) * S(s)| < 1 \quad \text{For all frequencies (NP)} \quad [3]$$

$$|Wi(s) * T(s)| < 1 \quad [4]$$

(|Wp(s) * S(s)| + |Wi(s) * T(s)|) < 1 For all w (RP) [5] to have good tracking of reference signal and good rejection of disturbance signal S(s) =0, T(s) =1, If and only if |L(s)| >>1. To prevent the propagation of measured noise to the error and output signal

S(s)=1,T(s)=0, If and only if |L(s)|<<1, Therefore |L(s)| >>1, is required at low frequency and |L(s)| <<1, is required at high frequency.

III. Robust stability of SISO time delay systems

The robust performance of SISO time delay system, in terms of weight sensitivity function derives a condition that will guarantees the system remains in stable for all Internal perturbations in the uncertainty set and To determine the stability of the uncertain closed loop system the following assumptions are considered, 1. L is stable, 2.Nominal closed loop system is stable i.e. if controller frequency response remains within the allowed region at all frequencies than RS and RP satisfied for all uncertain plants in the set. The most significant factor is that, the selection of system sensitivity weight function. The PID controller design procedure and tuning methods are discussed in section IV. Simulated results for different systems are presented in section V and conclusions from results are in section VI.

IV. PID Controller Design

$$G = \exp((-0.6 * s) * (-0.6s + 1) / ((s - 1) * (2 * s + 1))) \quad [6]$$

G = transfer function of 2rd order time delay system. The step response of time delay system, when supply the unit step input signal,

Table 1.1, Time domain specifications

Rise Time	5.0815
Settling Time	10.2157
Settling min	0.9012
Settling Max	0.9998
Overshoot	0
Undershoot	5.6249
Peak	0.9998
Peak Time	19.2496

Therefore at the initial stage step signal is in reverse direction and from the bode plot GM=9.17dB (at w=1.06 rad/sec), PM = -180deg (at w=0 rad/sec), Gain margin of the system is positive and Phase margin of the system is negative, Therefore the given system is unstable. A suitable PID controller design is essential that provide a better performance for robustness.

$$C(s) = K_p + K_i * \left(\frac{1}{s}\right) + K_d * s \quad [7]$$

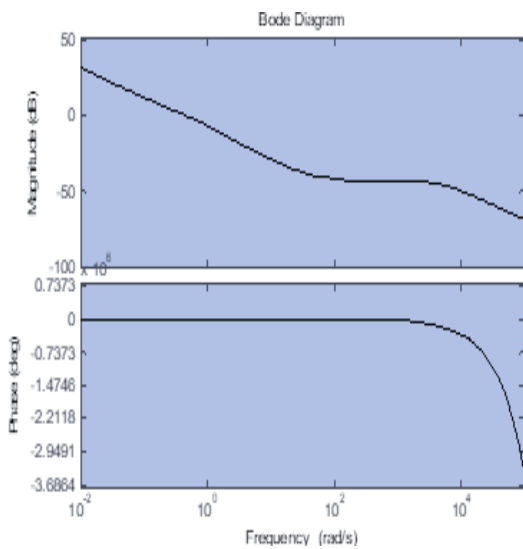
K_p, k_i, K_d are proportional, integral and derivative constants. The Z-N tuning method, $K_p = 0.834$, $K_i = 0.313$, $K_p = 0.216$, the controller parameters are used to obtain a controller transfer function.

$$L(s) = G(s) * C(s) \quad [8]$$

$$C(s) = 0.834 + 0.313 * \left(\frac{1}{s}\right) + 0.216 * s \quad [10]$$

$$L(s) = \exp(-0.6 * s) * \frac{((-72.11 * s^3 - 3450 * s^2 + 1883) / (2 * s^4 + 10003 * s^3 + 5000s))}{s^3 + 5000s} \quad [11]$$

System with PID controller, $BW = 0.4432$, $GM = 5.7$ dB (at $w = 0.893$ rad/sec), $PM = 47.4$ deg (at $w = 0.459$ rad/sec). From fig (1), Both GM and PM are positive; therefore the system is stable in margin



Fig(1), Bode plot of loop transfer function.

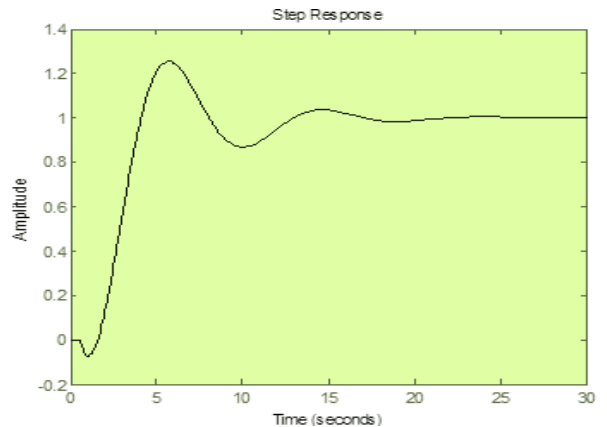


Fig (2). Step response $(L/(1+L))$

CRC Bode approach for time delay system are used to measure robust performance, in closed loop stability the sensitivity function must satisfy the each RHP - Z of transfer function G.

$$\text{Transfer function of Sensitivity} = 1 / (1+L) \quad [12]$$

where L is loop transfer function

$$\text{Complementary sensitivity function} = L / (1+L) \quad [13]$$

And the procedure for Performance weight function is attained. The performance weight functions W_p & W_i are

selected based on the sensitivity and inverse sensitivity of the given system, selection of performance weights, for closed loop stability the sensitivity function must satisfy the each RHP -Z of process that is

$$\| W_p * S \| \geq | W_p * M_{zpi} | \quad [14] \quad M_{zpi} = \prod_{i=1}^{N_p} (|Z + P_i|) / (|Z - P_i|) \text{ and}$$

$$W_p(s) = (s/M + w_b) / (s + w_b * A) \quad [15]$$

Where,

M = gain of high frequency disturbances

A = gain of low frequency control signal

w_b = bandwidth (or) cut off frequency.

This weight stresses low frequency performance and it specifies a minimum band width, a maximum peak of $|S|$ is less than M and steady state offset is less than A ($A < 1$) and bandwidth always greater than frequency.

$$W_p = (0.9 * s + 0.5) / (s + 1) \quad [16]$$

$$W_i = (s + 0.05) / (s + 1) \quad [17]$$

V. Results

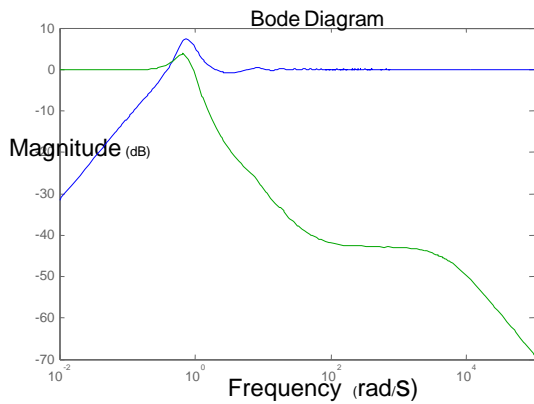


Figure (3) , bode mag(S,T)

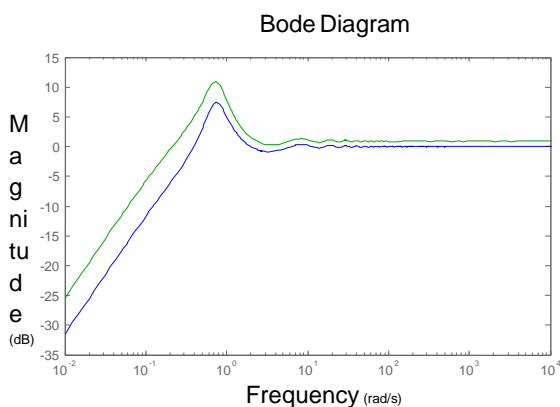


Figure (4), bodemag(S, S*1/Wp)

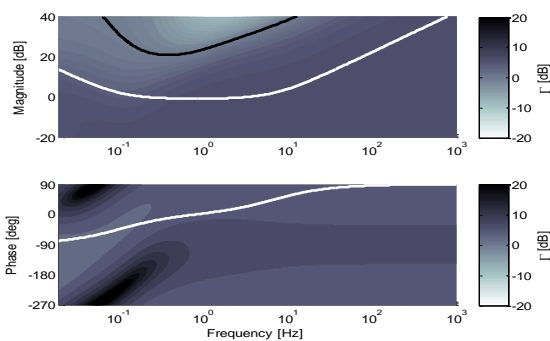


Figure (6) CRCBode plot

The Contours are not passing through the forbidden region Figure (6), Therefore the system is robustly stable with designed controller ($r < 1$) at all frequencies and uncertainty

Hence the SISO time delay system is robustly stable and also nominally stable at all uncertainty

VI. Examples

$$1..G(s) = 9 * \exp(-0.06 * s)/(s + 1)^2$$

$$2. G(s) = (0.3 * s)/((s + 3) * (s^2 + 2 * s + 3))$$

$$3. G(s) = 10 * \frac{\exp(-2.6*s)}{(s^2+0.9s+1)}$$

By using the tuning methods Kp, Kd, Ki all gain constants of PID controller is determined and this parameters are responsible for unit step response of loop transfer function (L) and the performance weights are selected as per the required bode stability of sensitivity functions and inverse sensitivity functions, ZN method is most used PID tuning online method And the PID controller Kp, Kd, Ki values with their performance weights of respected systems are tabulated.

Transfer function	PID controller parameters	Performance weight functions
1. $G(s) = 9 * \exp(-0.06 * s)/(s + 1)^2$	$Kp=1;$ $Kd=0.025;$ $Ki=1.022;$	$W1 = \text{tf}([5 \ 0.9],[1 \ 0.003]);$ $W2 = \text{tf}([0.1 \ 0],[20 \ 1]);$
2. $G(s) = \frac{\exp(-0.3 * s)}{(s + 3) * (s^2 + 2 * s + 3)}$	$Kp = 0.32;$ $Ki = 3.43;$ $Kd=0.02$	$W1 = \text{tf}([1],[1 \ 0.3]);$ $W2 = \text{tf}([0.1],[0.7 \ 1]);$
3. $G(s) = 10 * \frac{\exp(-2.6*s)}{(s^2+0.9s+1)}$	$Kp=0.001;$ $Kd=0.001;$ $Ki=0.0138;$	$WP = \text{tf}([8.5 \ 1],[4 \ 0.6]);$ $W1 = \text{tf}([0.125 \ 1],[0.5 \ 1]);$

Table 1.2.controller parameters and performance weight functions

VII. Results

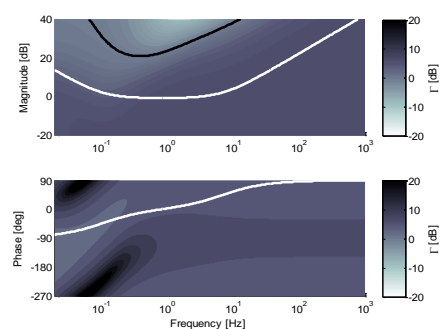


Figure (7) , $G(s) = 10 * \frac{\exp(-2.6*s)}{(s^2+0.9s+1)}$

In figure (7), the contours are not passing through the forbidden region of CRC bode plots therefore required NM, RS, RP are obtained.

NM –nominal model

RS – robust stability

RP – robust performance

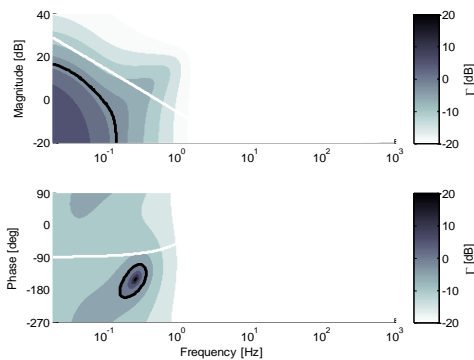


Figure 8), $G(s) = 8.9 * \exp(-0.06s)/(s + 1)^2$

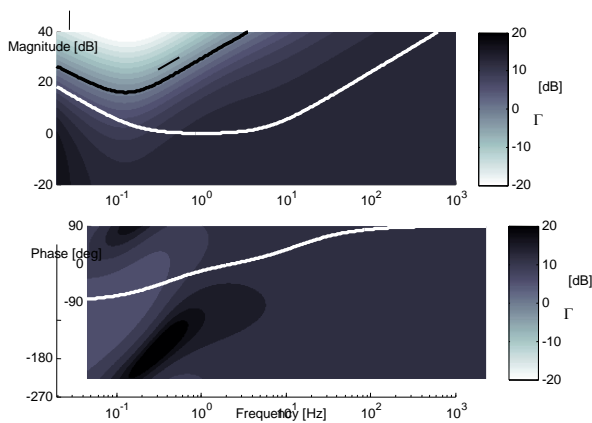


Figure 9), $G(s) = \frac{\exp(0.3 * s)}{(s + 3) * (s^2 + s + 3)}$

IX. CONCLUSIONS

The time delay systems exhibit undesired response for a input step signal, the proper selection of controller parameters the performance of system changes from unstable region to stable region and robust performance index is observed with the help of CRC Bode (i.e. contours of bode plot) thus, all contours travel outside of the forbidden regions of the CRC Bode therefore a robust control conceivable with the help of designed performance weights.

This approach is guarantees the robust stability and better performance of the system under uncertainty conditions and which satisfies the all constraints at all frequencies.

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