Analysis of Natural Frequency of a Four-Wheeler Passenger Car by Combined Rectilinear and Angular Modes an Analytical Approach

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Abstract - *Comfortability of the vehicle is most important for the passengers. Automobile and aerospace industries are spending millions of rupees on research work to reduce the vibrations in vehicles. The present research work shows the results of two modes of natural frequencies with respect to variation in mass as well as stiffness of front and rear suspension system of four-wheeler passenger cars. The results shows that the increase in mass as well as stiffness values of suspension system increases the two modes of natural frequencies of vibration of four-wheeler passenger cars. Every mechanical system has its own natural frequency, also called as fundamental frequency. When the systems are put in practical applications, many external forces will act over the system and due to which the systems will vibrate with forced frequencies. If the fundamental frequency and the forced frequency matches then resonance condition will occur leading to partial or full failure of the systems. So, avoiding the resonance as well as reducing the vibration in four-wheeler passenger cars is a challenging task for engineers to have a vehicle with more comfortability for the passengers.*

Key Words: *Natural frequency, Mode shapes, Passenger Car, Stiffness, Mass*

1.INTRODUCTION

Every mechanical body or system has its own natural frequency of vibration or it is also known as the fundamental frequency of vibration. Once the mechanical systems are put into practical applications different kinds of forces will act over the system causing forced vibrations in the system. The natural frequency of the system should not become equal to forced frequency. If the natural frequency of the system becomes equals to forcing frequency, then resonance will occur leading to complete or partial system failure. Since at resonance condition the system vibrates with very high or infinite amplitude. So, analysis of natural frequency of any mechanical system plays a very important role. In our present work determination of natural frequency of vibration of a four-wheeler passenger care is done by combined rectilinear and angular modes by analytical method. A four-wheeler passenger car with 2000kg mass is considered for the analysis. Equation of motion are derived and finally the two modes of natural frequencies are found out. Also, the variation of two modes of frequencies with varying stiffness of front and rear suspension system are analyzed. Also, the effect of varying mass of vehicle on the natural frequency of vibration of the four-wheeler passenger car is analyzed. Up gradation of comfortability of the vehicle is the most significant for the passengers. This will be done by the reduction of vibration within the vehicles. Vibration analysis finds its application in automotive industries, aerospace industries and many engineering fields. The automobile and aeronautical industries intensively work on the vibration analysis procedures and also on type of materials to achieve the objective of comfort of the passengers and also the durability and their life [1]. The essential ride model of a passenger car has suspension points at each of the four wheels. The wheels or axles have the mass. The vehicle designer tries to keep the body bounce resonant frequency as less as possible however this can be obtained as a result of constraints on the stiffness of the suspension imposed by the available range of suspension travel. Sport cars normally have higher natural frequencies and more damping due to considerations about their performance in handling [2]. Automotive vehicles are the first medium of transportation within the world and should consider the highest levels of safety and comfort for the passengers. The health and safety of the drivers and passengers within an automotive structure are often judged by finding out the dynamic interaction between the occupants and moving automotive. Dynamic investigations are quite common in numerous transportation industries like railways, naval, aviation and automobile sectors, wherever moving structure and its occupants are exposed to unwanted vibration. The essential objective of research and its findings is to judge and reduce the amount of vibration by designing the system or sub-system in the extreme operative conditions. Unwanted vibration within the automotive structures is produced by the uncontrolled and random excitation which may cause the resonance and huge displacement on the automotive car seat and occupant human bodies. Dynamic simulation analysis of the car seat and human body is known as a very complicated and non-linear phenomenon and hence, an entire analysis of all the non-linear parameters is required to carry out an effective simulation to determine the level of vibration within human body and car seat [3]. The vehicle suspension forms very important system for an automobile. This will often help to support the engine, vehicle body and passengers and at the same time absorbs the shocks arising due to roughness of the road. The normal arrangement consists of supporting the chassis through springs and dampers, by the axle. The engine and also the body of the vehicle are fitted to the chassis rigidly. Hence, the chassis along with the body of the vehicle, engine and the passengers may be considered to be one unit only. The springs and dampers which connect the axle and chassis have important role in absorbing shocks and keeping chassis affected to a minimum level [4]. Vibration evoked by the irregularities on the road surface and imbalance of the tire / wheel is transmitted to the vehicle body through the suspension, a vibration transmission system. Engine vibration is additionally transmitted to the vehicle body through the suspension system via the drive shaft [5]. When talking about the suspension system, spring rates and damping levels are the parameters that are usually used. Each elastic object or material will have a certain speed of oscillation which will occur naturally when there are zero outside forces or damping applied. This natural vibration happens solely at a precise frequency, referred as the natural frequency. Natural frequency is that frequency at which a system tends to oscillate in the absence of any driving or damping force. The motion pattern of a system oscillating at its own natural frequency is called the normal mode vibration if each parts of the system move sinusoidally with that same frequency. If the oscillating system is driven by an external force at the frequency at which the amplitude of its motion is largest and almost close or equal to a natural frequency of the system, this frequency is called as the resonant frequency. If a spring that is subject to a vibratory motion and is close to its natural frequency the spring will begin to surge. This case is extremely undesirable since the life of the spring can be reduced as excessive internal stresses developed. The operative characteristics of the spring are also seriously affected. For many springs subject to low frequency vibrations surging is not a problem. However, for high frequency vibrating applications it is necessary to ensure, in the design stage, that the spring natural frequency is fifteen to twenty or more times the maximum operating vibration frequency of the spring [6]. The pneumatic tyre perceives the road unevenness and roughness and transmits them to the suspension elements during the movement of a vehicle. The generated vibrations will enter into the passenger compartment through the connections between the suspension and the vehicle body. Properly choosing of elastic and damping characteristics of the suspension elements (shock absorber, spring, rubber mounts), and modal parameters of the metal elements (rods, shells, plates) can improve the car comfort in terms of noise and vibrations. This reduces the driver fatigue. Depending on the vehicle class, the vibration and noise levels in its cabin are different. The driver feels these vibrations in the seat, the floor panels and the steering wheel. [7].

2. COMBINED RECTILINEAR AND ANGULAR MODES OF VIBRATION OF A FOUR-WHEELER PASSENGER CAR

In actual practice, sometimes, the body is subjected to combined rectilinear and angular motions. For example, in case of vehicle, when brakes are applied on moving vehicle, two motions of vehicle occur simultaneously. One motion is rectilinear(x) and other motion is angular (θ). This type of combined motion in the system is due to the fact that C.G. of vehicle and centre of rotation do not coincide. Consider a car of mass m and moment of inertia I supported on two suspension systems with stiffness K_1 and K_2 as shown in figure 1

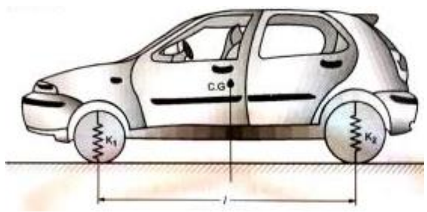

Fig1: Four-Wheeler Passenger Car

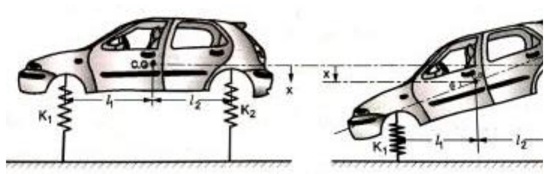

Fig.2: Equilibrium and displaced position

Let at any instant the vehicle is displaced through a rectilinear distance x and angular displacement θ as shown in fig1. Since θ is small therefore springs K_1 and K_2 are compressed through a distance $(x - l_1 \theta)$ and $(x + l_2 \theta)$ respectively. The free body diagram of the system is shown in figure 2

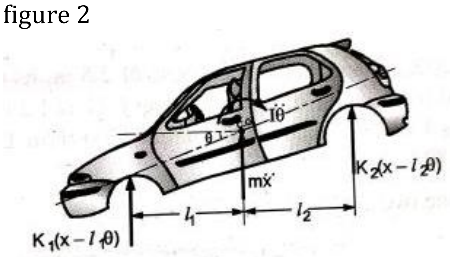

Fig 3: Free Body Diagram

The differential equation of motion for the system are

$$mx+ K_1 (x- l_1 \theta) + K_2 (x+ l_2 \theta) = 0$$

$$l\, \theta -K_1 (x- l_1 \theta)\, l_1 + K_2 (x+ l_2 \theta)\, l_2= 0$$

$$m\ddot{x}+(K_1+ K_2)\, x - (K_1 l_1 - K_2 l_2)\, \theta =0$$

$$l\, \ddot{\theta} - (K_1 l_1- K_2 l_2)\, x + (K_1 l_1^2 + K_2 l_2^2)\, \theta =0$$

The equations have both the term x and θ hence they are called the coupled differential equation. These equations indicate that system has rotary as well as translatory motion.

Case 1 – when $K_1 l_1 = K_2 l_2$

when $K_1 l_1 = K_2 l_2$ the equation the equations of motion become, $m\ddot{x}+(K_1+K_2)x = 0$ & $I\ddot{\theta} + (K_1 l_1^2 + K_2 l_2^2)\theta = 0$ From these equations it is seen that rectilinear and angular motions can exist independently. Such equations are called as uncoupled differential equations. The two natural frequencies of system are

$$\omega_{n1} = \sqrt{(k_1 + k_2)/m} \quad \& \quad \omega_{n2} = \sqrt{(l_1^2 + l_2^2)/I}$$

Case 2 – When $K_1 l_1 \neq K_2 l_2$

The Solution for x and θ under steady state condition are $X = X \sin \omega t$, $\theta = \Phi \sin \omega t$, Where, X and Φ are amplitude of rectilinear and angular motion respectively.

$\ddot{x} = - X \omega t \sin \omega t$ & $\ddot{\theta} = - \Phi \omega^2 \sin \omega t$

$-mX \omega^2 \sin\omega t + (K_1+ K_2) X\sin\omega t – (K_1 l_1 - K_2 l_2) \Phi\sin \omega t = 0$

$-mX \omega^2 + (K_1+ K_2) X– (K_1 l_1 - K_2 l_2) \Phi = 0$

$(K_1+ K_2 - m\omega^2) X = (K_1 l_1 - K_2 l_2) \Phi$

$$\frac{X}{\Phi} = \frac{(K_1 l_1 - K_2 l_2)}{(K_1 + K_2 - m\omega^2)}$$

$-I \Phi\omega^2 \sin \omega t – (K_1 l_1 - K_2 l_2) X \sin \omega t + (K_1 l_1^2 + K_2 l_2^2) \Phi \sin \omega t = 0$

$-I \omega^2\Phi – (K_1 l_1 - K_2 l_2) X + (K_1 l_1^2 + K_2 l_2^2) \Phi = 0$

$(K_1 l_1^2 + K_2 l_2^2 - I \omega^2) \Phi = (K_1 l_1 - K_2 l_2) X$

$$\frac{X}{\Phi} = \frac{(K^1 l^{12} + K^2 l^{22} - I \omega^2)}{(K_1 l_1 - K_2 l_2)}$$

$$\frac{(K_1 l_1 - K_2 l_2)}{(K_1 + K_2 - m\omega^2)} = \frac{(K^1 l^{12} + K^2 l_2^2 - I \omega^2)}{(K_1 l_1 - K_2 l_2)}$$

$(K_1+ K_2 - m\omega^2) (K_1 l_1^2 + K_2 l_2^2 - I \omega^2) = (K_1 l_1 - K_2 l_2)^2$

$K_1^2 l_1^2 + K_1 K_2 l_2^2 - K_1 I\omega^2 + K_1 K_2 l_1^2 + K_2^2 l_1^2 - K_2 I\omega^2 - K_1 l_1^2$

$m \omega^2 - K_2 l_2^2 m \omega^2 + Im \omega^4 = K_1 l_1^2 - 2 K_1 K_2 l_1 l_2 + K_2^2 l_2^2$

$I m \omega^4 – (K_1 I+ K_2 I+ K_1 l_1^2 m + K_2 l^2{}_2 m) \omega^2 + K_1 K_2 l_1^2 + K_1$

$K_2 l_2^2 + 2 K_1 K_2 l_1 l_2 = 0$

$Im\omega^4 – (K_1 I+K_2 I+ K_1 l_1^2 m + K_2 l^2{}_2 m) \omega^2+K_1 K_2 (l_1 + l_1)^2 = 0$

$$\omega^4 - \left(\frac{K_1}{m} + \frac{K_2}{m} + \frac{K_1 l_1^2}{I} + \frac{k_2 l_2^2}{I}\right)\omega^2 + \frac{k_1 k_2 (l_1 + l_2)^2}{Im} = 0$$

$$\omega^4 - \left(\frac{K_1 + K_2}{m} + \frac{K_1 l_1^2 + K_2 l_2^2}{I}\right)\omega^2 + \frac{K_1 K_2 (l_1 + l_2)^2}{Im} = 0$$

$\omega^4 – B\omega^2 + C = 0$

$B = \left(\frac{K_1+K_2}{m} + \frac{K_1 l_1^2 + K_2 l_2^2}{I}\right)$

$C = \frac{K_1 K_2 (l_1 + l_2)^2}{Im}$

$\omega^2 = \frac{+B \pm \sqrt{B^2 - 4C}}{2}$

$\omega^2{}_{n1} = \frac{1}{2}[B - \sqrt{B^2 - 4C}]$

$\omega^2{}_{n2} = \frac{1}{2}[B + \sqrt{B^2 - 4C}]$

3. NATURAL FREQUENCIES RESULTS BY ANALYTICAL METHOD

Table1: The natural frequency results of vibrations with constant mass and varying stiffness

K1	K2	L1	L2	K	M	I	ωn1	ωn2
450	500	2	2	1.2	1000	1440	0.9726	1.6257
500	550	2	2	1.2	1000	1440	1.0523	1.6611
550	600	2	2	1.2	1000	1440	1.0988	1.7428
600	650	2	2	1.2	1000	1440	1.1433	1.8207
650	700	2	2	1.2	1000	1440	1.1862	1.8955
700	750	2	2	1.2	1000	1440	1.2276	1.9674
750	800	2	2	1.2	1000	1440	1.2677	2.0368
800	850	2	2	1.2	1000	1440	1.3065	2.1039
850	900	2	2	1.2	1000	1440	1.3442	2.1689
900	950	2	2	1.2	1000	1440	1.3809	2.2320
950	1000	2	2	1.2	1000	1440	1.4166	2.2934
1000	1050	2	2	1.2	1000	1440	1.4515	2.3532
1050	1100	2	2	1.2	1000	1440	1.4855	2.4115

1100	1150	2	2	1.2	1000	1440	1.5188	2.4685
1150	1200	2	2	1.2	1000	1440	1.5514	2.5241
1200	1250	2	2	1.2	1000	1440	1.5833	2.5785
1250	1300	2	2	1.2	1000	1440	1.6145	2.6319
1300	1350	2	2	1.2	1000	1440	1.6452	2.6841
1350	1400	2	2	1.2	1000	1440	1.6753	2.7354
1400	1450	2	2	1.2	1000	1440	1.7049	2.7857
1450	1500	2	2	1.2	1000	1440	1.7340	2.8351
1500	1550	2	2	1.2	1000	1440	1.7626	2.8837
1550	1600	2	2	1.2	1000	1440	1.7907	2.9315
1600	1650	2	2	1.2	1000	1440	1.8184	2.9785
1650	1700	2	2	1.2	1000	1440	1.8457	3.0247
1700	1750	2	2	1.2	1000	1440	1.8726	3.0703
1750	1800	2	2	1.2	1000	1440	1.8991	3.1152
1800	1850	2	2	1.2	1000	1440	1.9252	3.1595
1850	1900	2	2	1.2	1000	1440	1.9510	3.2031
1900	1950	2	2	1.2	1000	1440	1.9765	3.2462
1950	2000	2	2	1.2	1000	1440	2.0016	3.2887
2000	2050	2	2	1.2	1000	1440	2.0265	3.3307

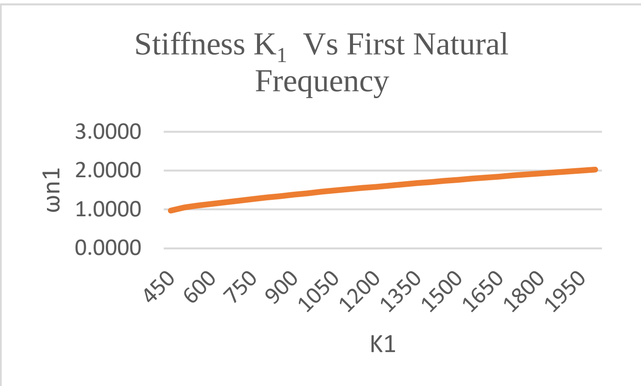

Fig:4 Stiffness K_1 Vs First Natural Frequency

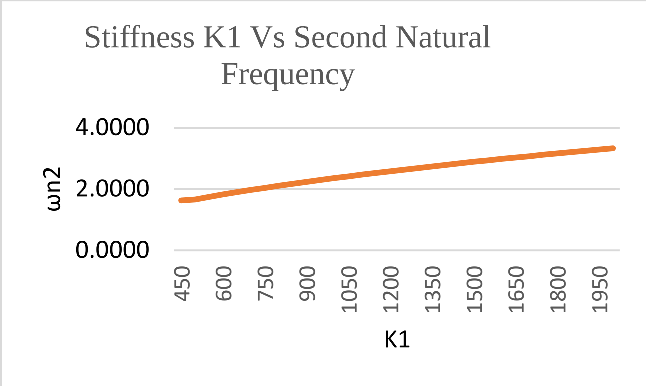

Fig:5 Stiffness K_1 Vs Second Natural Frequency

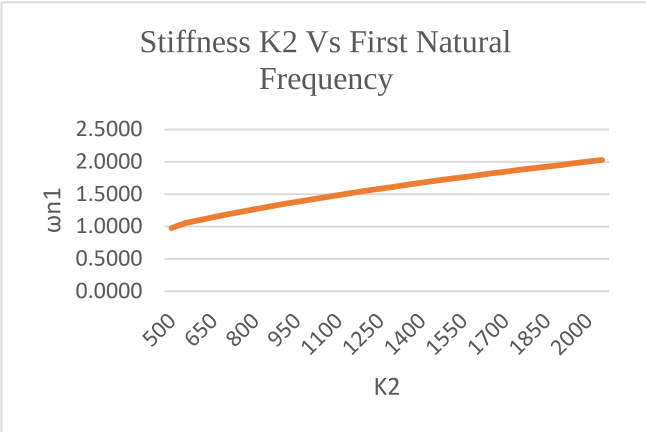

Fig:6 Stiffness K_2 Vs First Natural Frequency

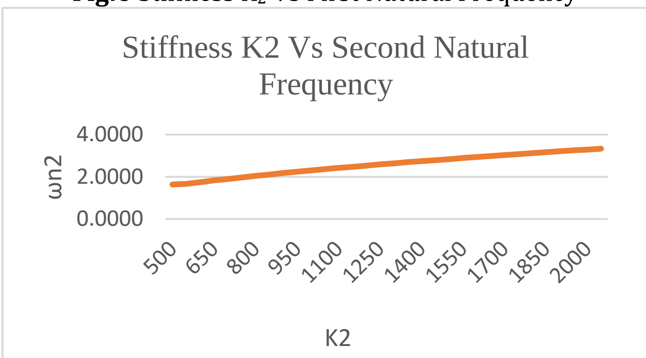

Fig:7 Stiffness K_2 Vs Second Natural Frequencies

The table 1 shows the results of two natural frequencies of vibration or fundamental frequencies of vibration of four-wheeler passenger cars calculated by means of using the two natural frequency equations (ω_{n1} & ω_{n2} equations) derived from the equation of motion of four-wheeler passenger cars. The total mass of the passenger car considered here is 1000kg. L1 and L2 have same value as 2m. Front and rear suspension system stiffness K1 & K2 varied from 450 N/mm to 2000N/mm and 500N/mm to 2050N/mm respectively.

The variation of front suspension system stiffness K_1 with ω_{n1} & ω_{n2} are shown in figure4 & figure 5 respectively. As the stiffness of front suspension system K_1 of the four-wheeler passenger car increases the two modes of natural frequencies ω_{n1} & ω_{n2} also increases. As the stiffness value K_1 of front suspension system increased from 450 N/mm² to 2000 N/mm² the first natural frequency ω_{n1} value increased from 0.9726 rad/sec to 2.0265 rad/sec. As the stiffness value K_1 of front suspension system increased from 450 N/mm² to 2000N/mm² the second natural frequency value varied from 1.6257 rad/ sec to 3.3307 rad/sec.

The variation of rear suspension system stiffness K_2 with ω_{n1} & ω_{n2} are shown in figure6 & figure7 respectively. As the stiffness of rear suspension system K_2 of the four-wheeler passenger car increases the two modes of natural frequencies ω_{n1} & ω_{n2} also increases. As the stiffness value K_2 of rear suspension system increased from 500 N/mm² to 2050 N/mm² the first natural frequency ω_{n1} value increased from 0.9726 rad/sec to 2.0265 rad/sec. As the stiffness value K_2 of rear suspension system increased from 500 N/mm² to 2050 N/mm² the second natural frequency value varied from 1.6257 rad/ sec to 3.3307 rad/sec.

Table2: The natural frequency results of vibrations with varying stiffness and mass values

K1	K2	L1	L2	K	M	I	ωn1	ωn2
450	500	2	2	1.2	1000	1440	0.9726	1.6257
500	550	2	2	1.2	1050	1512	1.0128	1.6438
550	600	2	2	1.2	1100	1584	1.0338	1.6838
600	650	2	2	1.2	1150	1656	1.0527	1.7194
650	700	2	2	1.2	1200	1728	1.0698	1.7514
700	750	2	2	1.2	1250	1800	1.0854	1.7802
750	800	2	2	1.2	1300	1872	1.0995	1.8064
800	850	2	2	1.2	1350	1944	1.1125	1.8302
850	900	2	2	1.2	1400	2016	1.1244	1.8521
900	950	2	2	1.2	1450	2088	1.1354	1.8721
950	1000	2	2	1.2	1500	2160	1.1456	1.8906
1000	1050	2	2	1.2	1550	2232	1.1551	1.9078
1050	1100	2	2	1.2	1600	2304	1.1639	1.9237
1100	1150	2	2	1.2	1650	2376	1.1721	1.9385
1150	1200	2	2	1.2	1700	2448	1.1798	1.9523
1200	1250	2	2	1.2	1750	2520	1.1871	1.9652
1250	1300	2	2	1.2	1800	2592	1.1938	1.9773
1300	1350	2	2	1.2	1850	2664	1.2002	1.9887
1350	1400	2	2	1.2	1900	2736	1.2063	1.9995
1400	1450	2	2	1.2	1950	2808	1.2120	2.0096
1450	1500	2	2	1.2	2000	2880	1.2174	2.0191
1500	1550	2	2	1.2	2050	2952	1.2225	2.0281
1550	1600	2	2	1.2	2100	3024	1.2273	2.0367
1600	1650	2	2	1.2	2150	3096	1.2319	2.0448
1650	1700	2	2	1.2	2200	3168	1.2363	2.0525
1700	1750	2	2	1.2	2250	3240	1.2405	2.0599
1750	1800	2	2	1.2	2300	3312	1.2445	2.0669
1800	1850	2	2	1.2	2350	3384	1.2483	2.0736
1850	1900	2	2	1.2	2400	3456	1.2519	2.0799
1900	1950	2	2	1.2	2450	3528	1.2554	2.0860
1950	2000	2	2	1.2	2500	3600	1.2587	2.0919
2000	2050	2	2	1.2	2550	3672	1.2619	2.0974

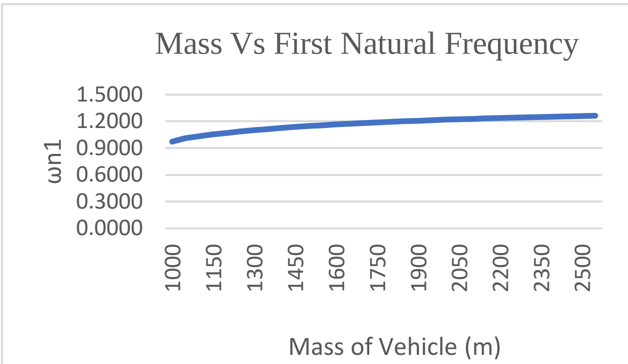

Fig: 8 Mass m Vs First Natural Frequency

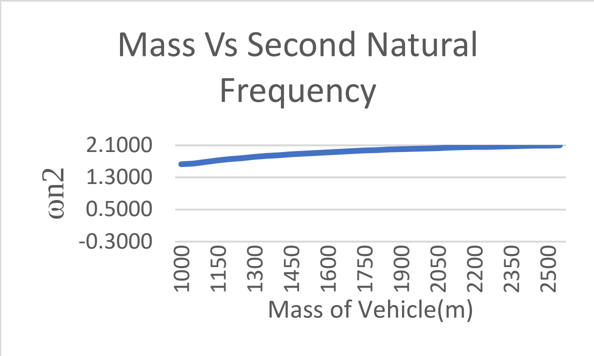

Fig:9 Mass m Vs Second Natural Frequency

Table 2 shows the results of two natural frequencies ω_{n1} & ω_{n2} of the four-wheeler passenger car with respect to variation of front and rear suspension system as well the variation in mass. The stiffness K1 value varied from 450 N/mm^2 to 2000 N/mm^2 and the stiffness value k2 varied from 500 N/mm^2 to 2050 N/mm^2 and the mass of the four-wheeler passenger car increased from 1000kg to 2550kg. The results of mass against two modes of natural frequencies ω_{n1} & ω_{n2} are plotted in figure8 and figure9 respectively. The results show that as the mass of the vehicle increases, the two modes of natural frequencies also increase.

4. CONCLUSION:

The present research work shows the results of two modes of fundamental frequencies ωn1 & ωn2 of vibration of a four-wheeler passenger car. The research work is done by using analytical method. The equations of motion for the four-wheeler passenger car in practical condition are devolved. The equations of motion are solved for calculating the two modes of fundamental frequencies ωn1 & ωn2. In first case the mass of vehicle is kept constant and the mas considered was 1000kg. The stiffness values of front suspension systems varied from 450 N/mm2 to 2000 N/mm2 and stiffness values of rear suspension systems varied 500 N/mm2 to 2050 N/mm2 . The first fundamental frequency values achieved are 0.9726 rad/sec to 2.0265 rad/sec and the second fundamental frequency values achieved are 1.6257 rad/ sec to 3.3307 rad/sec corresponding to the stiffness values of front and rear system. In second case the mass of vehicle is varied from 1000kg to 2550 kg corresponding to front suspension system stiffness values from 450 N/mm2 to 2000 N/mm2 and rear suspension system stiffness values from 500 N/mm2 to 2050 N/mm2. The results show that as the front and rear suspension system stiffness values increases with constant mass of vehicle the fundamental frequencies also increase linearly. Also, as the mass of vehicle along with front and rear suspension systems stiffness values increases the two the two modes of fundamental frequencies also increase linearly.

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