

Dynamic Response of Offshore Articulated Tower-Under Airy and Stokes Theories

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Abstract - Articulated tower is a compliant offshore structure that oscillates freely with environmental loads of wind and wave. Present study deals with the dynamic response of a single hinge Articulated tower. Hydrodynamic loading of the tower is computed using Morrison's equation. Acceleration and velocity terms used in Morrison equation is computed first by Airy theory and then by Stokes theory. Various responses of the Articulated tower are studied and a comparative analysis is done so as to see which of Airy wave and Stokes wave gives better response. Response of this Articulated tower is then studied under wave alone environment and wind+wave correlated environment. Fluctuating component of the wind is modelled by Simiu's spectrum, while the sea state is characterized by Pierson-Moskowitz (P-M) spectrum. Random waves and wind are simulated by Monte Carlo simulation technique. Response of Articulated tower is determined by time domain iterative method using Newmark's β integration scheme. It is concluded that Stokes wave gives improved response and presence of wind forces along with the waves amplify the responses significantly particularly at lower frequencies.

Key Words: P-M (Pierson-Moskowitz), A-F (Axial Force), A-T (Articulated Tower), (M-E) Morrison Equation, (A-T) Airy Theory, (S-T) Stokes Theory.

1. INTRODUCTION

Articulated Tower is a flexible compliant offshore structure which resist the environmental forces through the action of compliancy, which means that one single buoyant shell with sufficient buoyancy is used to restore moment against the lateral loads. Articulated towers are designed such that their fundamental frequency is well below the wave frequency to avoid dynamic amplification. Dynamic interaction of these towers with environmental loads (wind, waves and currents) acts to impart a lesser overall shear and overturning moment due to compliance to such forces. This compliancy introduces geometric nonlinearity due to large displacements, which becomes an important consideration in the analysis of articulated towers. Wind and wave loadings have a predominant role in the design of an offshore structure for a successful service and survival in harsh sea conditions. The flexibility of the new generation, Wind and wave loadings have a predominant role in the design of an offshore structure for a successful service and

survival in harsh sea conditions. compliant structures (articulated tower, guyed tower and tension leg platforms) give rise to natural periods ranging from 1 to 100 seconds. Such structures comply in the direction of environmental loads and results in an increase in their sensitivity to the dynamic effects of wind. Fixed structures will respond them in virtually static fashion. For fixed platforms, contribution of lateral wind loads for the design is only 10% of the global loads. While in case of compliant platforms, it increases to 25%.

1.1 TOWER RESPONSE UNDER AIRY AND STOKES THEORIES

A single hinged Articulated Tower of height 400m standing in 350m deep water has been chosen for present study. The platform characteristics are given in Table 1. Two different sea states i.e Moderate sea state and High sea state has been considered in the study. The characteristics of these two states are given in Table 2. As flexural deformations are negligible, hence re neglected in comparison to rigid body displacement. Responses of the tower have been obtained for two most widely used wave theories i.e Airy theory and Stokes theory. Random waves are represented by Pierson-Moskowitz spectrum. The code was developed using FORTRAN software. Time histories and Power Spectral Density Functions (PSDF's) are obtained for steady state condition of the tower. To obtain the steady state condition transition phase of oscillations which is roughly ten times the time period of the structure has been ignored. Time histories of responses are obtained at an interval of 0.7s using Newmark β time integration method. Statistical parameters such as maxima, minima, mean and standard deviation are obtained for Deck Displacement, Hinge Shear and Axial Force by using Airy and Stokes wave theory. Drag coefficient is taken as 0.6 and Inertia coefficient is taken as 2.0.

Sea state	Significant wave	Zero crossing
Description	height, H_s (m)	period, T_z (s)
Moderate sea state	4.51	7.38
High sea state	12.4	12.23

Table -1: Characteristics of different sea states

1.2 Advantage of Articulated Tower

- Very large fundamental sway period so dynamic amplification factor is much less than fixed structure.
- Enhanced “turnability” of periods of the system for a particular site. An adjustment of natural frequency could be assisted by set up of a ballast water chamber positioned above the connector joint.
- Articulate loading platforms are re-usable. Once an oil reservoir is depleted, it can be easily relocated to other field at a minimal cost.
- Articulated loading platforms are used as a portable offshore system for the moored tankers. Such configuration is particularly suitable for fields that have a limited production capability, or are too far off from refining to justify the laying of pipe line.

Articulated Towers around the World

Field	Type	Installation Year	Water Depth	Operator	Platform Function
Beryl	Articulated loading Tower	1975	117	Ben C. Gerwick	Loading Tower
Statford	Articulated Tower	1978	145	EHM	Loading and Mooring
Maureen	Gravity articulated tower	1982	490	Howard Doris Ltd.	Drilling and production
North East Frigg.	Single Hinge Articulated Tower	1983	150	Total E&P UK Ltd	Field control station & gas Prod
Garden Banks	Compliant articulated tower (CAT)	1998	501	Amerada Hess Corp.	Drilling and production

Need of Articulated Tower in Indian Scenarios

In India Oil has been found at the shallower depth usually up to 100-150 m. Articulated tower can be advantageously used for such a shallow depth. As depth of exploration increases the possibility of finding oil and gas in deeperwater increase. As depth of water increases, size of conventional fixed legplatform will cross their effective economic size making it unsuitable. So there arises a need of new structural systems to be effectively used in deeper water. One such solution is articulated Tower which take advantage of the effect of compliance, i.e., yield to the environmental forces. With the

discovery of Oil and Natural gas in the deeper water, there arises a need of economical solution of offshore structure. Conventional fixed leg platform is found to be economical for shallow depth. As depth of water increases, size of these platform will cross their effective economic size making it unsuitable. So there arises a need of new structural systems to be effectively used in deeper water. One such solution is articulated Tower which take advantage of the effect of compliance, i.e., yield to the environmental forces.

Reliability of Articulation System

Various environmental loads such as Wind load, wave load, ocean currents etc are continuously applied on the articulate towers. These oscillating stresses can cause various type of damage to the structure such as metal fatigue which is caused due to stress concentration of various shear and axial forces.

Various researchers have done remarkable work in combat the problem of designing articulate joint.

Author	Year	Salient Features
Chassy et al	1971	<ul style="list-style-type: none"> • He gave the details of the universal joint for the ELFOCEAN tower. • The study was carried out considering the wear tear and risk of the collapse of pipe walls passing through the joint
Sedillot et al	1982	<ul style="list-style-type: none"> • He designed a ball type universal joint. • Field investigations for fatigue life were performed for few years. Based on tests, the minimum life of the articulation system was estimated as 200 years.

Objective

- To do a study of dynamic response of Articulated Tower by two widely used wave theories, i.e Linear Wave Theory (Airy Theory) and Stokes Theory (5th order theory) to conclude which of the two theory gives improved responses.

2. METHODOLOGY

In the present work firstly a nonlinear dynamic analysis of the said structure under waves / earthquake has been carried out for its time domain responses using Lagrangian approach which has the capability of equating kinetic and potential energies of the system to the rotational degrees of freedom. The random waves have been simulated by Monte-

Carlo technique represented by Modified PM spectra. Modified Morison's equation has been used for estimation of hydro-dynamic loading. Water particle kinematics has been governed by Airy's linear wave theory and Stoke's fifth order theory. Result obtained from both the theory are compared with each other. To incorporate variable submergence, Chakraborty's correction [20 & 21] has been applied. Seismic inputs have been applied using Northridge, Imperial valley CA, Duzce, Turkey spectra. Stability assessment has been carried out using concept of minimum potential energy and two-dimensional phase plots. Analysis of the response of Articulated Tower to fluctuating Wind and Wave Forces is done by a time domain iterative procedure which includes the structural as well as forcing nonlinearities.

Modelling of the Articulated Tower

Modelling of Articulated Tower will be nonlinear. Three type of nonlinear matrix will be formed. Stiffness matrix which consist of fluctuating Buoyancy component, Mass matrix which consist of two type of mass, one is structural mass and other being the added mass due to the motion of Tower and third matrix will be the Damping matrix. The tower structure is idealized by replacing its mass distribution with discrete masses located at the centroid of a series of small cylindrical elements of equivalent diameter D_i representing inertia, added mass and buoyancy. All forces are assumed to act at these centroids. In submerged part the forces act is weight, inertia, buoyancy and fluid force while wind force acts on exposed area.

Linear Wave Theory

Linear wave theory or Airy theory is used where surface of water is not changing or is too small to notice. Let us define a potential function Φ which is proportional to velocity of fluid. Consider a progressive wave with water surface elevation depicted by cosine curve. It is mandatory to analyse the effects of surface waves on the structures, either using a single design wave chosen to represent the extreme storm conditions in the area of interest, by use of statistical representation of the waves during extreme storm conditions.

Free Water surface elevation is given by

$$\eta = \frac{H}{2} \cos(kx - \omega t) \tag{1}$$

And the corresponding velocity potential is

$$\frac{-H \omega \cosh k(h+z)}{2 k \sinh kh} \sin(kx - \omega t) = \Phi \tag{2}$$

Morison Equation

Wave and current loading can be calculated by Morison equation as:

$$F_T = \frac{1}{2} C_D \rho_w D V [V] + \frac{1}{2} C_M \rho_w a \tag{3}$$

Where F_T is the total force, ρ_w is the density of water, C_D and C_m are the drag and inertia coefficient respectively, D is the diameter of member including marine growth, V is the velocity and a is the acceleration.

First term is the drag component and second is the inertia component.

Total Drag = Drag component + Inertia Component.

The force $dF(t)$ due to wave on differential section of length d_{si} of the cylinder is made up of two components namely inertia force component which is proportional to the normal component of the fluid particle acceleration and drag force which is proportional to the square of the normal component of the fluid particle velocity thus:

$$dF(t) = \left[\frac{\pi}{4} D^2 \rho_w C_m (\ddot{u}) + \frac{1}{2} \rho_w D C_D \dot{u} |\dot{u}| \right] d_{si} \tag{4}$$

$$dF(t) = \left[\frac{\pi}{4} D^2 \rho_w C_m (\ddot{u}) + \frac{1}{2} \rho_w D C_D (v_c + \dot{u}) |v_c + \dot{u}| \right] d_{si} \tag{5}$$

$$dF(t) = \left[\frac{\pi}{4} D^2 \rho_w C_m (\ddot{u} - \dot{x}) + \frac{1}{2} \rho_w D C_D (\dot{u} - \dot{x}) |\dot{u} - \dot{x}| \right] d_{si} \tag{6}$$

Follows:

Lagrange's equation

The general form of Lagrange's equation is:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}_i} \right) - \frac{\partial T}{\partial \theta_i} + \frac{\partial V}{\partial \theta_i} = Q_{\theta_i} \tag{7}$$

where T , V and Q_{θ_i} represents the kinetic energy, the potential energy and the generalized force, respectively.

Deck Displacement

Deck displacement under moderate sea state

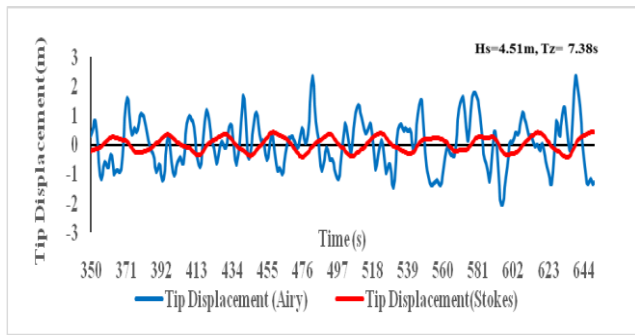


Fig -1: Time history of Deck Displacement by Airy and Stokes wave theories under moderate sea state.

Sea State	Statistics	Wave only	
		Airy	Stokes
Moderate	Maximum	2.33	0.47
	Minimum	-2.05	-0.50
	Mean	0.02	0.02
	Standard Deviation	0.83	0.25
High	Maximum	5.28	3.03
	Minimum	-5.59	-3.01
	Mean	-0.12	0.01
	Standard Deviation	2.08	1.26

Table -2: Statistical comparison of Deck Displacement values by Airy and Stokes wave theories.

Hinge Rotation

Hinge rotation under moderate sea state

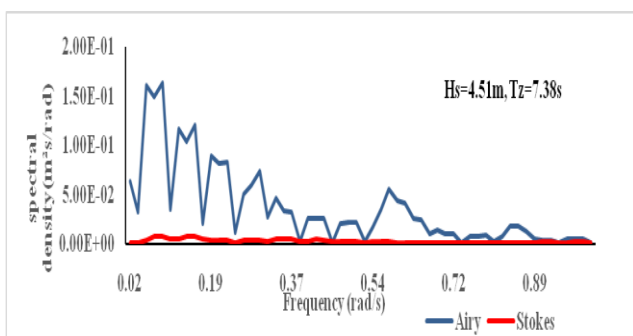


Fig -2: PSDF of Deck Displacement by Airy and Stokes wave theories under moderate sea state.

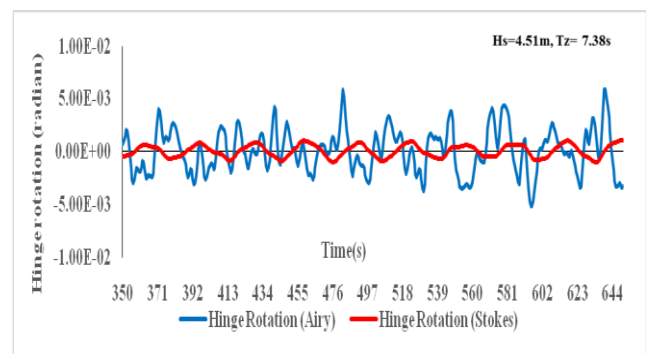


Fig -5: Time history of Hinge rotation by Airy and Stokes wave theories under moderate sea state.

Hinge rotation under high sea state

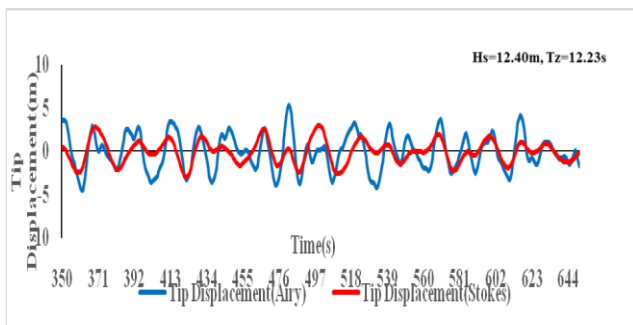


Fig -3: Time history of Deck Displacement by Airy and Stokes wave theories under high sea state.

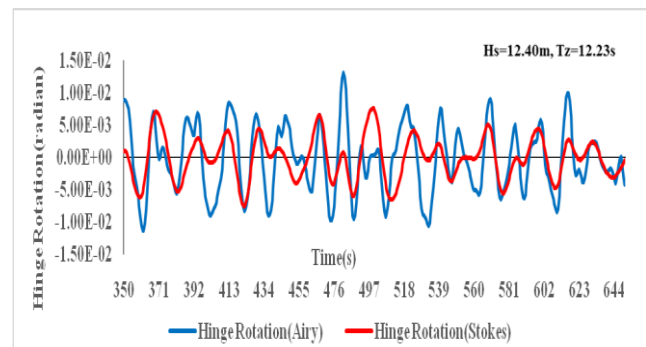


Fig -6: Time history of Hinge rotation by Airy and Stokes wave theories under high sea state.

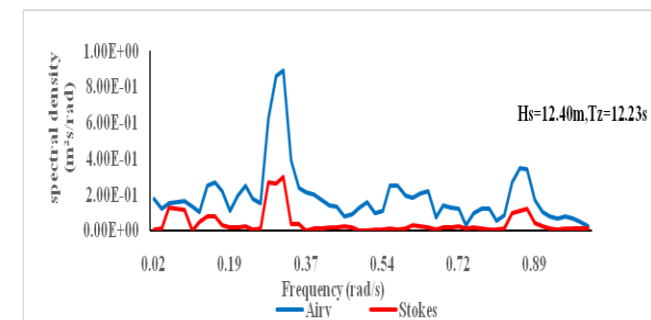


Fig -4: PSDF of Deck Displacement by Airy and Stokes wave theories under high sea state.

Hinge Shear

Hinge shear under moderate sea state

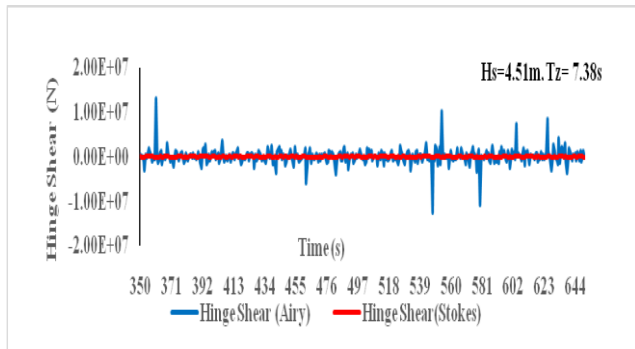


Fig -7: Time history of Hinge Shear by Airy and Stokes wave theories under moderate sea state.

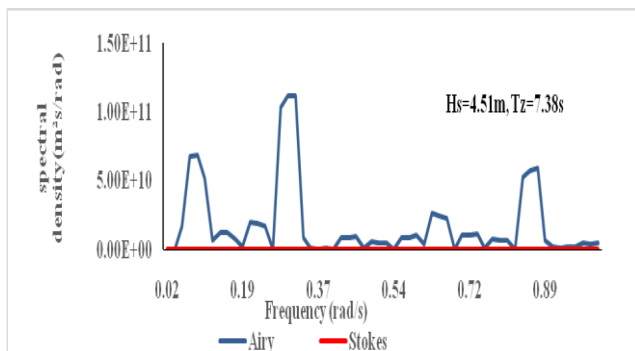


Fig -8: PSDF of Hinge Shear by Airy and Stokes wave theories under moderate sea state.

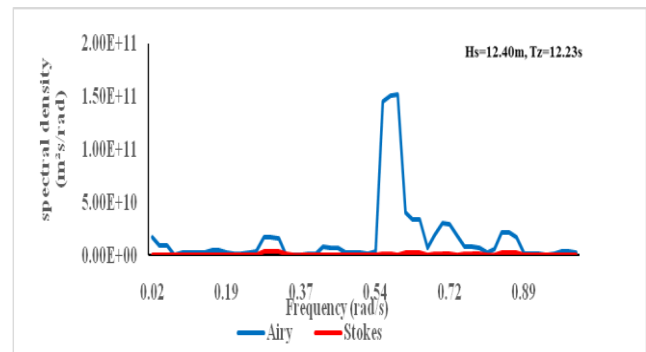


Fig -10: PSDF of Hinge Shear by Airy and Stokes wave theories under high sea state.

Sea State	Statistics	Wave only	
		Airy	Stokes
Moderate	Maximum	1.29E+07	1.25E+05
	Minimum	-1.25E+07	-1.65E+05
	Mean	-7.26E+03	-1.47E+04
	Standard Deviation	1.70E+06	3.66E+04
	Maximum	6.49E+06	9.76E+05
High	Minimum	-7.58E+06	-6.41E+05
	Mean	1.35E+05	1.93E+04
	Standard Deviation	1.39E+06	2.02E+05

Table -3: Statistical comparison of Hinge Shear values by Airy and Stokes wave theories.

Hinge shear under high sea state

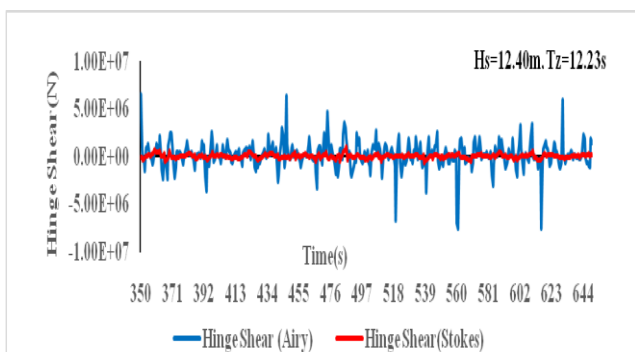


Fig -9: Time history of Hinge Shear by Airy and Stokes wave theories under high sea state.

Axial Force: Axial force under moderate sea state

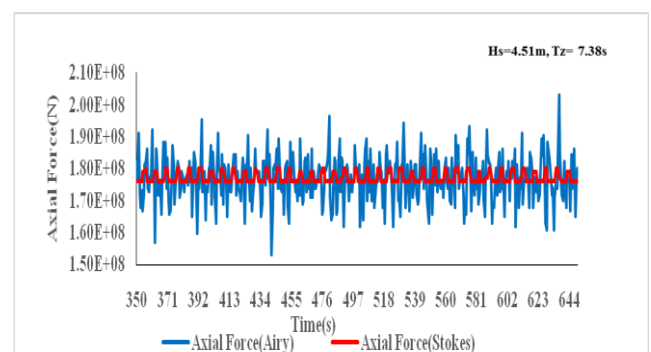


Fig -11: Time history of Axial Force by Airy and Stokes wave theories under moderate sea state.

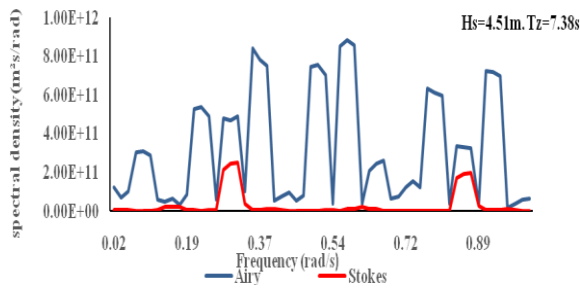


Fig -12: PSDF of Axial Force by Airy and Stokes wave theories under moderate sea state.

Axial force under high sea state

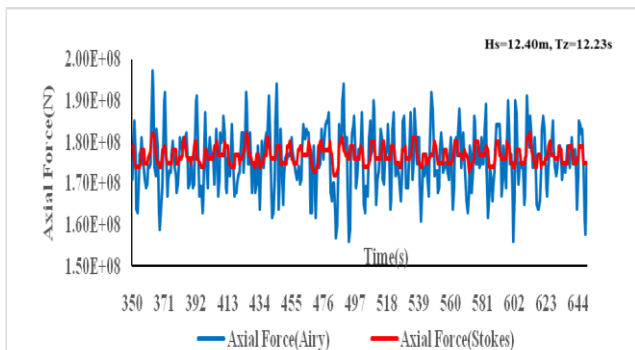


Fig -13: Time history of Axial Force by Airy and Stokes wave theories under high sea state.

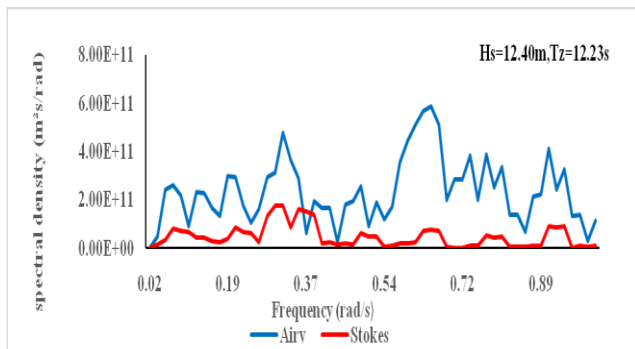


Fig -14: PSDF of Axial Force by Airy and Stokes wave theories under high sea state.

Minimum	1.56E+08	1.72E+08
Mean	1.76E+08	1.77E+08
Standard Deviation	7.20E+06	2.10E+06

Table -4: Statistical comparison of Axial Force values by Airy and Stokes wave theories.

3. CONCLUSIONS

In the present study Articulated offshore tower has been investigated. First the hydrodynamic loads on the Articulated tower is estimated by using the Airy theory, which is a linear wave theory and is frequently used where surface is not changing or change is too small to notice. Then the hydrodynamic loads are estimated by more rational Stokes 5th order theory which takes into account the surface variation of waves. Results of time histories, PSDF's and statistical values shows that the responses obtained by using Stokes theory are improved responses and are in line to the actual responses to a large extent.

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Sea State	Statistics	Wave only	
		Airy	Stokes
Moderate	Maximum	2.03E+08	1.80E+08
	Minimum	1.53E+08	1.76E+08
	Mean	1.77E+08	1.77E+08
	Standard Deviation	7.14E+06	1.69E+06
High	Maximum	1.97E+08	1.82E+08

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